Thesis for the Degree of Ph.D.

Faculty of Engineering

University of Glasgow.

ELECTRODE SYSTEMS FOR THE MEASUREMENT OF PERMITTIVITY AND POWER FACTOR OF SHEET INSULATING MATERIAL, AT POWER FREQUENCIES AND AT HIGH VOLTAGE STRESSES.

Presented by

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CONTENTS.

FOREWORD.

The following paper consists of a collection of work done by the author in the Electrical Engineering Department of the Royal Technical College, Glasgow, under the direction of Professor S. Parker Smith, D.Sc., M.I.E.E., A.M.Inst.C.E. The work was begun in 1932 and certain parts have been published in the Journal of the Royal Technical College, in 1934 and 1937. The illness of the author in the latter year contributed to the delay in presenting this collection of the whole work. As mentioned on p.95, the author had the assistance of students of the College for certain of the experimental measurements.

The work relates to investigations of the errors introduced into measurements of dielectric loss-angle and permittivity of sheet insulating material at 50 cycles per second by the use of various electrode forms. Attempts have been made to evolve suitable easily applied electrode systems to facilitate the testing of such specimens with greater accuracy than is obtainable with the flat plate electrodes widely used in the past. In the course of the investigation, a new type of electrode system was evolved, employing liquid films of controlled thickness between the specimen of insulating material and the electrode conducting surfaces.

INTRODUCTORY.

A knowledge of the losses occurring in insulating materials when electrically stressed is of great importance in assessing their ability to withstand the stresses to which they will be subjected in practice. The breakdown voltage of many materials is related to the dielectric loss angle of the material, on account of the phenomena of breakdown due to thermal instability. Typical results illustrating this relation, giving measurements made on two thicknesses of synthetic-resin laminated boards are shown in Table A. The measurement of the dielectric loss angle of insulating materials therefore constitutes an example of "non-destructive"* material testing.

The theory is well known. If δ, the dielectric loss-angle, is the divergence of the phase of the charging current I of a capacitor from quadrature with the voltage V, the power dissipated in the dielectric material is

$$P = VI \cos\left(\frac{\pi}{2} - \delta\right)$$
$$= VI \sin \delta$$

For normal values of δ met with, the power may be expressed as

$$P \doteq VI \tan \delta$$
$$\text{or } V^2 \omega C \tan \delta,$$

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* See Reference (1)

TABLE A.

Specimen.	Power factor.	Breakdown voltage, kV (minute value)
A 1	0.070	51.5
2	0.072	54.5
3	0.083	46.2
4	0.087	43.2
5	0.123	35.4
B 1	0.104	31.0
2	0.113	23.0
3	0.145	20.5
4	0.630	11.5

tan δ being commonly referred to as the "power-factor" of the material. Thus the measurement of the power factor, and incidentally the permittivity χ, is effected by determining the loss-angle and the capacitance of the resulting capacitor.

An error, however, is introduced by the electrodes applied to the specimen for the purpose of the measurement. The choice of suitable electrodes which will give a reasonable accuracy in the determinations and yet be easily applied, is difficult. For high-precision testing, and for the investigation of the behaviour of solid dielectrics, mercury electrodes are suitable.*

For works and routine testing there is need for electrodes, which, while not possessing the highest accuracy of laboratory methods, are more easily manipulated than mercury electrodes. Referring to industrial measurements, Churcher, Dannatt and Dalgleish+ say ... "variations of 20% in power factor and 10% in permittivity cannot be considered of great importance." Flat brass electrodes have, in the past, been extensively used in such works tests. Thus in connection with the B.S. Specification 234/1925‡ "Ebonite for Electrical Purposes, Hague⊕ stated ..... "For ordinary works tests the

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* See References (2), (3) and others.
+ Ibid (4)
‡ Ibid (5a); since superseded by B.S.S. 234/1933 (Reference (5b)).
⊕ Ibid (6a)

specimen may consist of a 15 cm. diameter disc of the insulating sheet to which flat brass electrodes are attached". Hartshorn and co-workers state * ..... "they (brass plates) may be used for samples which are thick, ($\frac{1}{4}$" or more) and flat."

The uncertainties attaching to measurements made with such electrodes, particularly at high voltage stresses, are however well known. For example, Fig 1 shows measurements of the power factor of synthetic-resin varnish-paper boards using, in all cases, flat brass plate electrodes. Curves (a) give the figures as measured by the M.-V. Electrical Company Limited, whilst (c) and (b) relate to measurements on the same specimens at the Royal Technical College using electrodes with and without an additional 56 lb. load on the upper electrode. The variations are seen to be such that except for the roughly approximate works tests, the use of flat-plate electrodes is not advisable, particularly if the electrodes are not applied under pressure.

The following paper gives the results of investigations into the errors involved in the use of flat-plate electrodes under various conditions; the employment of modifications of this method using tin-foil electrodes applied in different ways; and finally, the development of a flat-plate electrode system using liquid films between the electrodes and the specimen.

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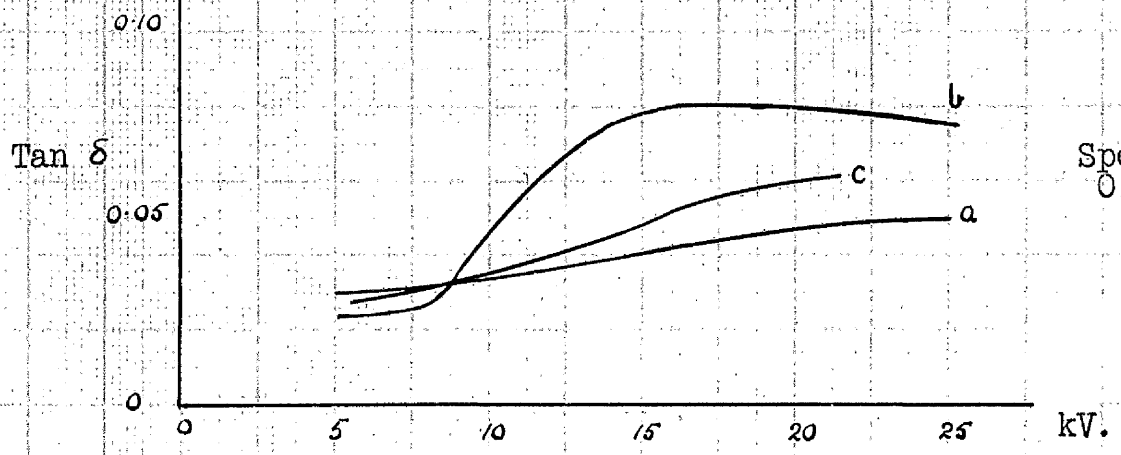
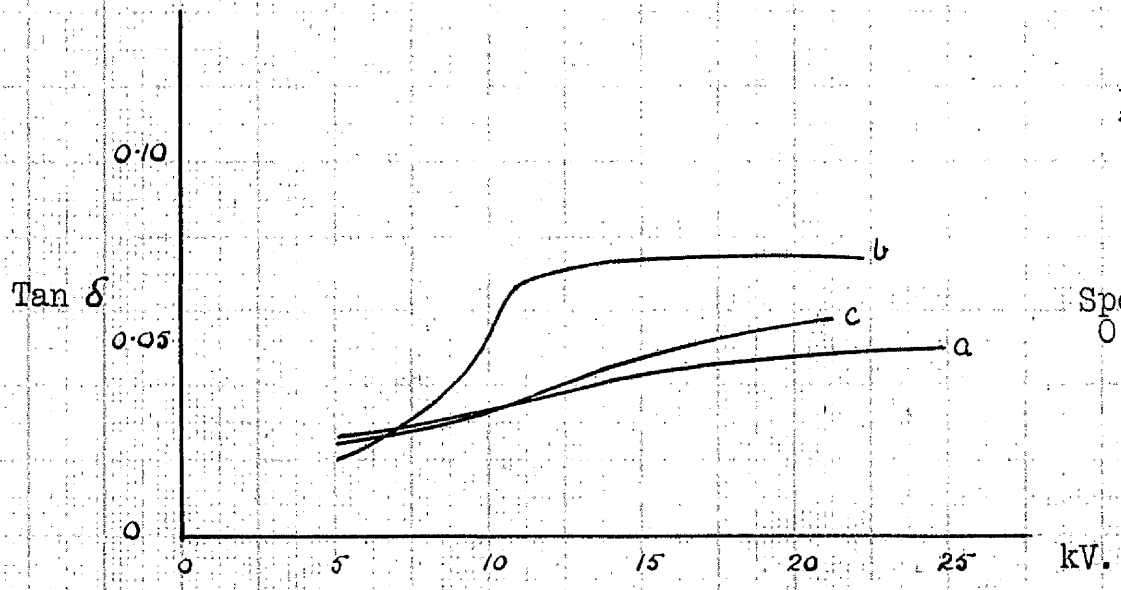
* See Reference (7).

POWER FACTOR MEASUREMENTS ON VARNISH PAPER BOARDS

(a) M.-V. Research Laboratory
(b), (c) R.T.C. Laboratory

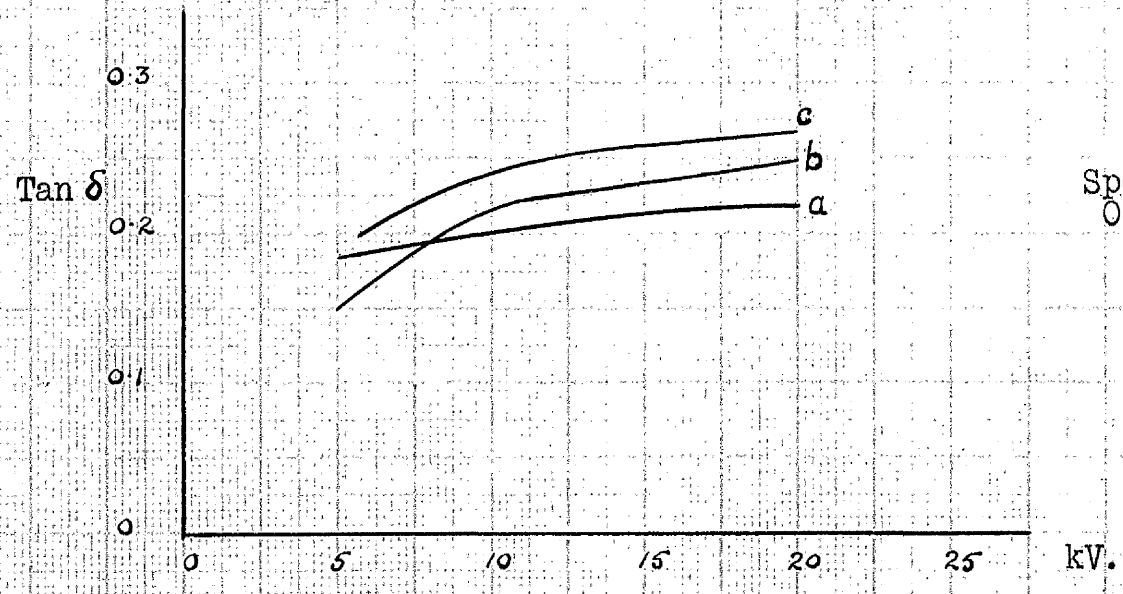


Specimen A
0.311 in.



Specimen B
0.311 in.

Fig. 1



Specimen C
0.191 in.

THE NATURE OF THE CONTACT EFFECT.

The fact that the apparent constants of a piece of insulating material, measured by making this material the dielectric of a capacitor, are found to depend on the electrodes used, implies that some value of series impedance must be ascribed to each type of contact in the particular conditions under which it is used. This effect may range from one which is almost entirely resistive to one in which it is practically capacitive. Barlow * even suggests that some substances such as celluloid can only be classed as insulators by virtue of their high contact resistance. It is clear, however, that, as Hartshorn † states, many anomalies in dielectric testing are explicable by the effect of the effective resistance or capacitance between electrode and dielectric. Hartshorn ‡ also suggests that contact impedance is a property of the actual dielectric-electrode interface, and that the idea of a perfect contact involving no potential drop may be fundamentally unsound, as there is a definite discontinuity in electrical properties at the interface. Such discontinuity is an inherent property of the dielectric and not merely an impedance which can be ascribed to defective electrodes.

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* See Reference (8)
† See Reference (9)
‡ Ibid (10)

A considerable amount of published data is available on contact effects, but it is difficult to correlate some of the information. That the behaviour of a given contact varies widely with the frequency and voltage at which the test is made has been well established. Here correlation will be attempted only insofar as it is necessary for the present purpose.

PREVIOUS INVESTIGATIONS OF CONTACT EFFECTS.

An extensive investigation into contact effects has been made by Churcher, Dannatt and Dalgleish * (E.R.A. report L/T 31). Illustrating the variations obtainable, they quote values of apparent power factor of varnished cloth, at 1000 cycles per second with different electrodes. (Table B).

In order to estimate the contact resistance with direct current, measurements of resistance between circular electrodes were made with the electrodes progressively displaced from the position in which they were directly opposite one another. Some materials, e.g. ebonite, gave regularly increasing resistances, suggesting that the insulation resistance is greater than the contact resistance. Other materials gave a smaller rate of increase of resistance, suggesting a preponderance of contact resistance; or, in the case of laminated materials, a smaller resistance along, than across, the laminae. As this latter ratio is about 3.5 for paxolin, the method is obviously useless for laminated materials.

Another direct-current method used by the above workers, that of measuring the potential drops along a cylindrical specimen, gave, by

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* See reference (4)

TABLE B.

Electrode	Measured power factor
Mercury	0.184
Tinfoil	0.173
Copper plates	0.134

extrapolation, contact resistances as follows:- slate, approx. $0.8 \times 10^6 \Omega$ per cm^2; fibre, approx. $7 \times 10^6 \Omega$ per cm^2.

Alternating current measurements were made by the same authors on a sheet of insulating material machined to successively smaller thicknesses, and by extrapolation of the results to the case of zero thickness, the contact resistance was estimated. This was shown to lie between 7×10^6 and $75 \times 10^6 \Omega$ per cm^2 for a range of materials. Another a.c. method, similar to the second d.c. method mentioned above, gave the magnitude and phase of the voltage across the contact and, therefrom, the constants of the contact impedance with mercury, graphite, tinfoil, copper and other electrodes. The results of a large number of tests showed that the contact-impedance phase-angle was nearly always in excess of 0.5 and often as high as 0.8 - 0.9. The values of contact resistance show a range of the order of $0.2 \times 10^6 \Omega$ to $200 \times 10^6 \Omega$ per cm^2.

The fact that the contact impedance of mercury on low power factor specimens is of the same order as that on those of high power factor precludes the explanation of contact impedance on the basis of the assumption that contact is intermittent and high density transverse currents are set up.

The authors incline to a view (cf. Hartshorn*) that the contact impedance is a property of the interface.

Finely divided graphite and sprayed zinc both gave lower contact impedances than mercury.

Assuming typical values of equivalent series resistance and capacitance of $50 \times 10^6 \Omega$ and 200 $\mu\mu F$ per cm^2 at each contact, the errors in the measured value of permittivity and power factor are calculated by the authors as shown in Table C.

Colloidal graphite, in the form of "aquadag" - a colloidal suspension of graphite stabilised with ammonia - is widely used in insulation measurements+ and the above view of its properties as an electrode compared with those of mercury is confirmed by Hartshorn‡ and Schrader⊕. Dannatt and Goodall*, in tests on mica at frequencies of 60 and 800 cycles per second state that aquadag is reliable, but that uncertainties due to dirt occur with mercury. These tests, however, refer to a much higher grade of insulating material ($\tan\delta \doteq 0.0003$) than the majority of the materials considered in the present discussion.

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* See Reference (10)
\+ Ibid (11)
‡ Ibid (12)
⊕ Ibid (13)
* Ibid (3)

TABLE C.

Thickness cm	Overall Measured values		Percentage Error	
	P.F.	K	P.F.	K
1.0	0.8	20	-1	-1.8
0.1	0.8	20	-0.1	-18
1.0	0.4	10	-0.8	-0.9
0.1	0.4	10	-10	-9
1.0	0.05	5	-.7	-0.45
0.1	0.05	5	-6	-4.5

An investigation by Churcher, Dannatt and Dalgleish* (E.R.A. report L/T 45) gives comparative figures for flat plate, aquadag and mercury electrodes on medium grade insulating material at 50 cycles per second and at different voltage stresses. Satisfactory agreement between aquadag and mercury was obtained. Flat plates gave values of power factor which were initially low, but which increased rapidly with increased voltage stress.

Hartshorn, Ward, Sharpe and O'Kane+ give results of an extensive investigation made at frequencies of 0.8×10^3 to 5×10^6 cycles per second and which is, therefore, complementary to the two E.R.A. reports mentioned above. Their investigations cover the use of brass plates, aquadag backed with brass plates, tinfoil attached either by aquadag or vaseline, mercury, and copper plating. They found errors ($>$ 100 per cent) due to contact effects much greater than those suggested in E.R.A. report L/T 31.‡ They conclude that mercury⊕ or graphite backed with mercury, and tinfoil-vaseline# give satisfactory results and that graphite and flat plates give results that are usually too high and too low respectively.

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* See Reference (14)
\+ Ibid (7)
‡ Supra p.9.
⊕ See also Reference (2)
\# Tinfoil electrodes, described later, were also found to be satisfactory by the present author. See Reference (15).

Each of the foregoing investigations emphasise the complexity of the problem. Electrodes which appear to be quite satisfactory at a particular frequency and voltage stress are seen to be capable of giving anomalous results under other conditions. Table D is compiled from the published results of these investigations dealing with material in sheet form; only a small number of the results can be included here but the figures serve to illustrate the wide variations obtainable in the measured value of power factor.

REFERENCE	4	14						7					
TEST	1	2	2a	3	3a	4	4a	5	5a	5b	6	7	7a
MATERIAL	Fibre	Ebonite		Varnish paper board		Glass		Ebonite				Glass	
THICKNESS, mils	1000	31		31		64		250			-	169	
FREQUENCY	70	50	50	50	50	50	50	1.1×10^4	2.5×10^5	5×10^6	800	1000	10^6
VOLTAGE	165	-	-	-	-	-	-	-	-	-	-	-	-
STRESS, V/mil	-	5	90	5	45	5	45	-	-	-	-	-	-
ELECTRODES	Power Factor												
FLAT PLATE	-	0.0072	0.0116	0.0550	0.0563	0.0222	0.0512	0.0254	0.0289	0.0242	0.0056	0.0213	0.0086
FLAT PLATE, loaded	-	-	-	-	-	-	-	-	-	-	0.0060	-	-
MERCURY	0.69	0.0075	0.0072	0.062	0.0618	0.0277	0.0280	0.028	0.031	0.026	-	0.0256	0.0096
TINFOIL	0.69	-	-	-	-	-	-	-	-	-	-	-	-
TINFOIL & VASELINE	-	-	-	-	-	-	-	0.271	0.0306	0.0254	0.0069	0.0268	0.0096
TINFOIL ON RUBBER	-	-	-	-	-	-	-	-	-	-	0.0060	-	-
AQUADAG	0.69	0.0078	0.0085	0.0642	0.0648	0.0289	0.0289	0.028	0.0363	0.0415	-	-	-
AQUADAG & BRASS	-	-	-	-	-	-	-	-	-	-	-	0.0262	0.044
AQUADAG & TINFOIL	0.69	-	-	-	-	-	-	0.0303	0.033	0.0299	-	0.256	0.0094
AQUADAG & COPPER	0.69	-	-	-	-	-	-	-	-	-	-	-	-
AQUADAG & MERCURY	0.69	-	-	-	-	-	-	-	-	-	0.0071	0.0264	0.0101

TABLE D

DIELECTRICS IN SERIES.

As previously mentioned, the contact effect can be regarded as a series impedance. A consideration of the overall power factor of a particular dielectric and electrode system involves a consideration of the magnitude and phase angle of the impedance of each.

An imperfect dielectric may be represented by a circuit arrangement of a capacitance with resistance in series or in parallel, such that the combination has the same effective impedance and phase angle as the dielectric being represented. Figs. 2 and 3 show such arrangements, the loss angles being $\delta_s = \tan^{-1} \omega C_s R_s$ and $\delta_p = \tan^{-1} 1/\omega C_p R_p$ respectively.

Expressions for the equivalent overall capacitance and loss angle of a dielectric specimen and contacts may be obtained on the basis of the series or parallel arrangements.

Let C_D, C_c, C; K_D, K_c, K; R_D, R_c, and R be the equivalent series capacitance, permittivity, and resistance of the dielectric, contact and the combination respectively. The equivalent parallel values are denoted by C_D', C_c', C'; K_D', K_c', K'; R_D', R_c', and R'. Then, if δ_D, δ_c and δ are the loss angles of a particular case of dielectric,

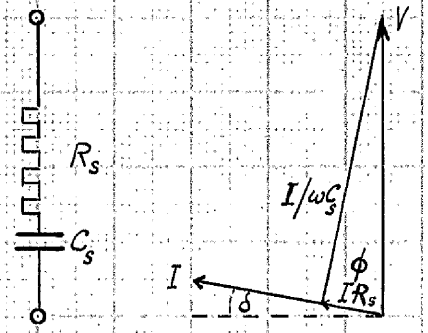


Fig. 2

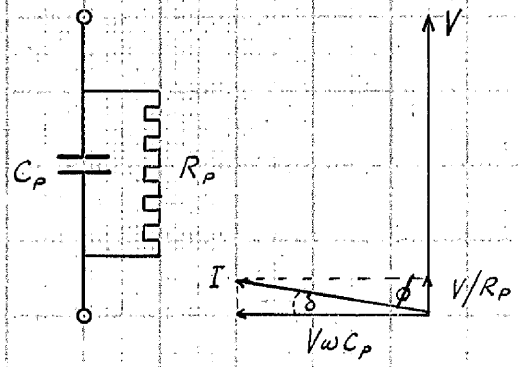


Fig. 3

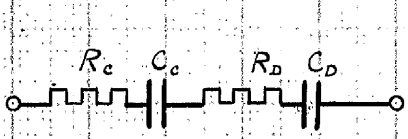
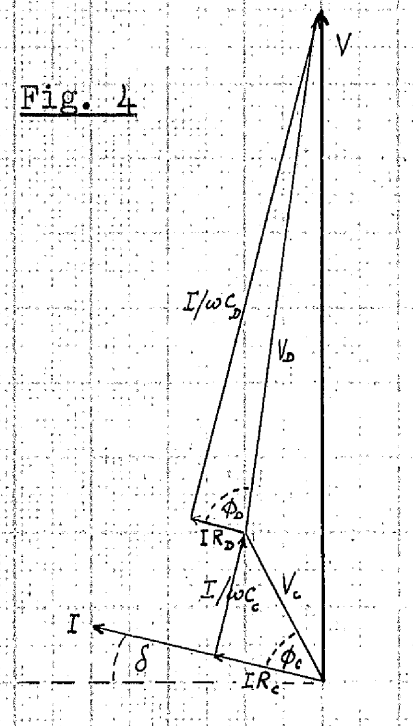


Fig. 4



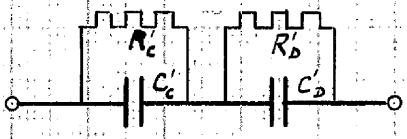
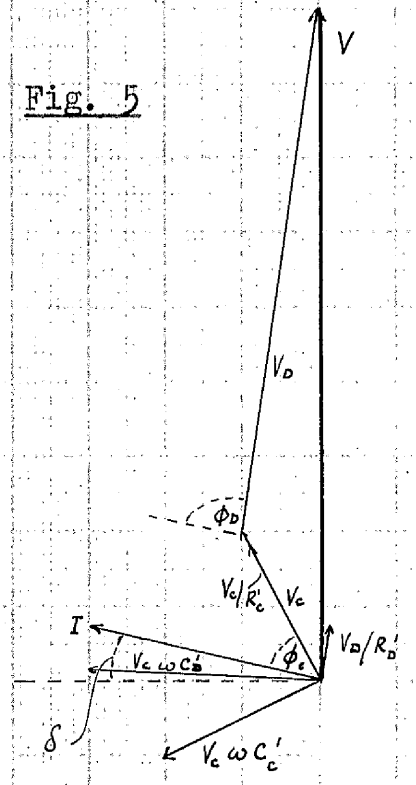


Fig. 5



contact and combination,

$$\omega C_D R_D = \tan \delta_D = 1/\omega C_D' R_D'$$
$$\omega C_c R_c = \tan \delta_c = 1/\omega C_c' R_c'$$
$$\omega C R = \tan \delta = 1/\omega C' R'$$

The series arrangement is shewn in Fig.4 from which it is seen that $C = C_c C_D/(C_c+C_D)$ .....(i)

and $\tan \delta = \omega C R$.

$$= \omega C_c C_D (R_c+R_D)/(C_c+C_D)$$
$$= \frac{C_c C_D}{C_c+C_D}\left(\frac{\tan \delta_c}{C_c} + \frac{\tan \delta_D}{C_D}\right)$$
$$= C_c \tan \delta_D/(C_c+C_D) + C_D \tan \delta_c/(C_c+C_D)$$

.....(ii)

On the basis of the parallel arrangement of Fig. 5 the equivalent <u>series</u> impedance $R - j/\omega C$ of the combination of specimen and contacts is given by

$$\left[\frac{R_c'}{1+\omega^2 C_c'^2 R_c'^2} + \frac{R_D'}{1+\omega^2 C_D'^2 R_D'^2}\right] - j\left[\frac{\omega C_c' R_c'^2}{1+\omega^2 C_c'^2 R_c'^2} + \frac{\omega C_D' R_D'^2}{1+\omega^2 C_D'^2 R_D'^2}\right]$$

Expanding this we have

$$C = C_c'\left[\frac{(1+\tan^2 \delta_c)(1+\tan^2 \delta_D)}{1+\frac{C_c'}{C_D'}+\frac{C_c'}{C_D'}\tan^2 \delta_c + \tan^2 \delta_D}\right]$$

.....(iii)

and

$$\tan\delta = \frac{C'_D \tan\delta_c (1+\tan^2\delta_D) + C'_c \tan\delta_D (1+\tan^2\delta_c)}{C'_c + C'_D + C'_c \tan^2\delta_c + \tan^2\delta_D \cdot C'_D} \qquad \ldots\ldots\ldots\ldots\text{(iv)}$$

From (iii) the equivalent <u>parallel</u> capacitance C' can be deduced if desired, but this is not essential, as $C' \doteq C$ (actually $C' = C/(1+\tan^2\delta)$).

Hartshorn, Ward, Sharpe and O'Kane* have obtained the following expressions for C' and tan δ,

$$C' = \frac{C'_D C'_c}{C'_D + C'_c} \; \frac{\left[1 + \frac{C'_c}{C'_c + C'_D}\tan^2\delta_c + \frac{C'_D}{C'_c + C'_D}\tan^2\delta_D\right]}{\left[1 + \left(\frac{C'_c}{C'_c + C'_D}\tan\delta_c + \frac{C'_D}{C'_c + C'_D}\tan\delta_D\right)^2\right]} \qquad \ldots\ldots\ldots\ldots\text{(v)}$$

$$\tan\delta = \tan\delta_D \; \frac{\left[1 + \tan^2\delta_c + \frac{C'_D}{C'_c}\left(\frac{\tan\delta_c}{\tan\delta_D} + \tan\delta_c \tan\delta_D\right)\right]}{\left[1 + \frac{C'_b}{C'_c} + \tan^2\delta_c + \frac{C'_D}{C'_c}\tan^2\delta_D\right]} \qquad \ldots\ldots\ldots\ldots\text{(vi)}$$

Inspection shows that (vi) is equivalent with (iv) deduced above.

Churcher, Dannatt and Dalgleish+ give expressions for the permittivity and phase angle of a dielectric in terms

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* See Reference (7)
\+ Ibid (4)

of C_c, the equivalent series capacitance in $\mu\mu F$ per cm^2 of the contact; R_c, R, the equivalent series resistance per cm^2 of the contact and the combination respectively; and κ the overall permittivity of the combination. Using these symbols, the overall capacitance per cm^2 is

$$\frac{\kappa}{0.91 \times 4\pi t_D} = 0.0885 \frac{\kappa}{t_D} \ \mu\mu F;$$ where t_D is the thickness of the specimen in cm,

$$\frac{\text{observed permittivity}}{\text{permittivity of material}} = \left[1 - \frac{0.0885\ \kappa}{C_c\ t_D}\right], \qquad \text{.....(vii)}$$

and

$$\frac{\tan \phi}{\tan \phi_D} = \frac{1 - \frac{R_c}{R}}{1 - \frac{0.0885\ \kappa}{C_c\ t_D}} \qquad \text{.....(viii)}$$

where $\phi = \left(\frac{\pi}{2} - \delta\right)$ and $\phi_D = \left(\frac{\pi}{2} - \delta_D\right)$.

Hartshorn*, in considering specimens consisting of n layers of thickness t_1, assumes $(n-1)$ separating air spaces of permittivity = 1 and thickness t_0, and gives

$$\frac{1}{\kappa} = \frac{n-1}{n} \frac{t_0}{t_1} + \frac{1}{\kappa_D} \qquad \text{.....(ix)}$$

and

$$\tan \delta = \tan \delta_D \Big/ \left(1 + \frac{n-1}{n} \kappa_D \frac{t_0}{t_1}\right) \qquad \text{.....(x)}$$

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* See Reference (12)

The calculation of $1/\kappa$ in (ix) assumes the overall thickness to be nt_1, whereas it is really $= nt_1 + (n-1)t_0$. (x) can be written as

$$\tan\delta = \tan\delta_D \Big/ \left(1 + \frac{\kappa_D t_{SPACES}}{t_D}\right) \qquad \text{.....(xi)} .$$

It is seen from (ii) and (iv) that if $\tan\delta_c = \tan\delta_D$, the value of power factor as measured, is the true value. $\tan\delta$ is less than or greater than $\tan\delta_D$ according as to whether $\tan\delta_c$ is less than or greater than $\tan\delta_D$. For the case of $\tan\delta_c = 0$, it is seen from (ii) that

$$\tan\delta = \frac{C_c \tan\delta_D}{C_c + C_D} \qquad \text{.....(xii)}$$

and from (iv)

$$\tan\delta = \frac{C'_c \tan\delta_D}{(C'_c + C'_D)\left(\frac{C'_D}{C'_D + C'_c}\tan^2\delta_D + 1\right)} \qquad \text{.....(xiii)}$$

The $\frac{C'_D}{C'_c + C'_D}\tan^2\delta_D$ term is of negligible importance and (xiii) thus reduces to

$$\tan\delta \doteq \frac{C'_c}{C'_c + C'_D}\tan\delta_D \qquad \text{.....(xiv)}$$

THE MEASUREMENT OF POWER FACTOR AND PERMITTIVITY.

Measurements of the impedance characteristics of capacitors are commonly made by an a.c. bridge method. The majority of the results quoted on pages 9 to 15 were so obtained, although for high frequencies the method of substitution in a resonant circuit * is also used.

In contradistinction to the case of wattmeter and calorimeter methods of power measurement, bridge methods using a detector tuned to the fundamental supply frequency do not + measure the losses due to harmonic frequencies. The impedance characteristics so measured refer to the fundamental frequency. The fundamental frequency impedance constants may or may not be the same as the constants at higher frequencies and may be modified by the latter, under certain conditions, e.g., by the heat produced by the harmonic currents.

The bridge network usually employed is that suggested by Schering ‡ and developed for loss measurements by Semm ⊕. Other networks ⁜ are in use for such measurements but the Schering bridge network is most common. The arrangement of the bridge network used in the present tests is shewn in Fig.6. The 50 cycle supply for the bridge network was obtained

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* See Reference (7)
+ Ibid (16), (17)
‡ Ibid (18)
⊕ Ibid (19)
⁜ Ibid (20)

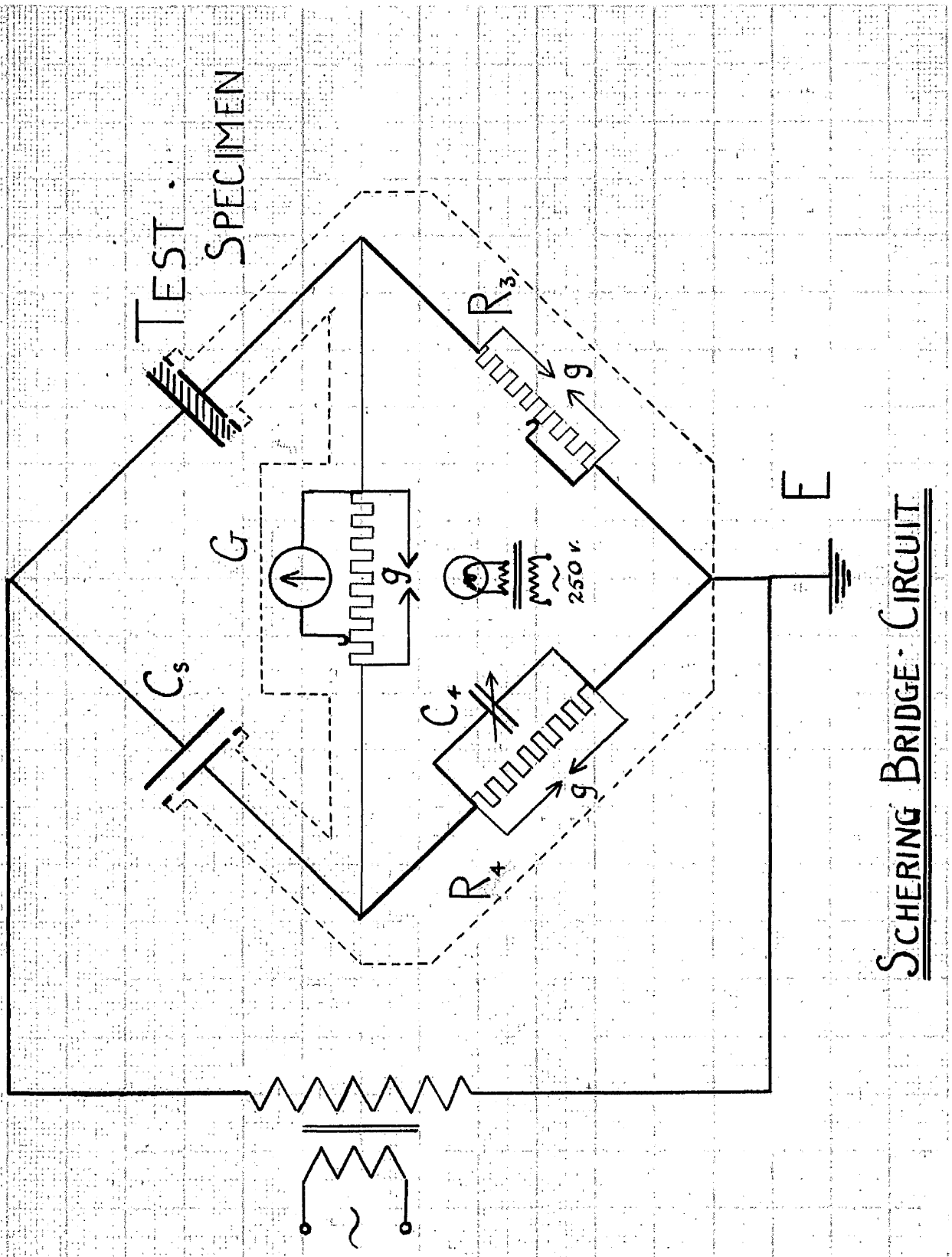
TEST SPECIMEN
R_3
g
G
g
~250 v.
E
C_s
C_x
g
R_x
SCHERING BRIDGE CIRCUIT

Fig.6

from a 100 kV transformer fed from a specially designed sine-wave alternator with a potentiometer-controlled field. The high-voltage side of the bridge consisted of a standard air capacitor C_s and the specimen under test complete with one of the types of electrodes described later. The standard air capacitor was of the compressed gas type which, working at a pressure of 85 lb. per square inch, was suitable for operation on a 100 k.v. (r.m.s.) circuit. Such a capacitor can be assumed to be loss-free, the impurity* due to dust being normally negligible. The bridge components were screened by an earthed shield as shewn by the dotted lines in Fig.6. The operator and the various components on the low-voltage side of the network were protected by safety gaps g, g, against danger from excess voltage in the event of the specimen under test breaking down. A variable resistance shunt was provided for the tuned 50-cycle vibration galvanometer.

The bridge components were as follows,

C_s , 106 $\mu\mu F$

R_3 , 0 - 6000 Ω , variable in steps of 0.01 Ω

R_4 , 318.4 Ω

C_4 , 0 - 11 μF, variable in steps of 0.001 μF.

The values of loss angle and effective series capacitance of specimen and electrodes together were

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* See References (21) and (22)

calculated from the following relations*,

$$\tan\delta = \omega R_4 C_4 = 0{\cdot}1\, C_4 \quad (C_4 \text{ in } \mu F),$$
$$C = C_5 R_4 / R_3 = 33800 / R_3 \ \mu\mu F.$$

The effective series permittivity of the specimen may be calculated from C as follows

$$K = 14{\cdot}4\, Ct / d^2$$

where d is the inside diameter of the "bridge" electrode guard-ring, in cm, and t is the thickness of the specimen.

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*Appendix, infra p.99.

Fig. 7a. Schering bridge and control bench.

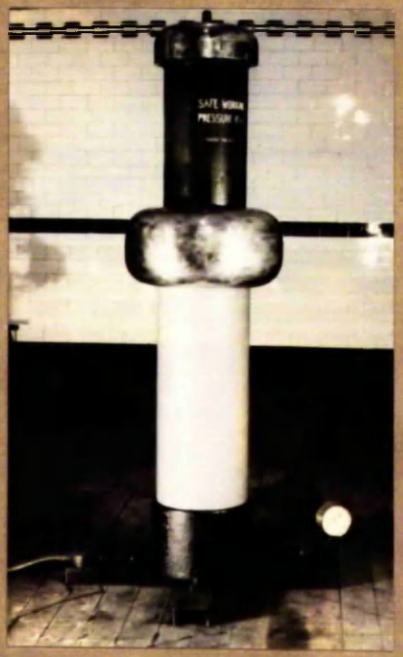

Fig. 7b. Standard air capacitor.

A. COMPARATIVE MEASUREMENTS MADE WITH AQUADAG, FLAT-PLATE AND TIN-FOIL ELECTRODES. *

It has been shown that for sheets of normal grade insulation, of thickness say 200 mils, the errors in the loss-angle and permittivity determined at industrial frequencies using mercury or graphite electrodes, are negligible, and that these types of electrodes or sprayed metal electrodes are desirable for accurate investigations of the properties of the dielectric and of contact effects. For commercial determinations, however, since the manufacturing methods often do not permit of a close control of the properties of the dielectric, this high degree of accuracy is not so essential in routine tests. It is for this reason that a consideration of the factors affecting the accuracy of power factor and permittivity measurements made with flat-plate electrodes was thought to be of interest, in spite of the inaccuracies known to be introduced by this type of electrode. Their ease of application is a strong inducement to their use where these inaccuracies can be tolerated, particularly if the errors can be reduced by suitable precautions.

The present section of the paper deals with the possibility of such reductions, and with the use of tin-foil as an electrode material.

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* A summary of this section was published in the Journal of the Royal Technical College, 1934; see Reference (15).

APPARATUS AND METHODS.

Aquadag Electrodes.

The results of the E.R.A. reports L/T 45* and L/T 50+ show that the differences of observed permittivity and power factor at industrial frequencies when using electrodes of mercury and divided graphite in the form of a colloidal solution, "Aquadag", are negligibly small. Accordingly, in the present series of tests, electrodes of aquadag were used as a standard by which the new types of electrodes were compared. The application of the aquadag was effected as follows. After cleaning the surfaces of the dielectric, the centres of the high-voltage and "bridge" electrodes were marked to be opposite one another. With these centres, circles were drawn with a fairly dilute solution of aquadag in distilled water, using an ink compass, to mark the inner and outer edges of the guard-ring gap and the outer edges of the electrodes. The areas so marked out were then filled in by means of a camel hair brush, using as strong a suspension of aquadag as possible. This method is illustrated in Fig.8(a).

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* See Reference (14)
\+ Ibid (23)

Fig. 8a. Application of aquadag electrodes.

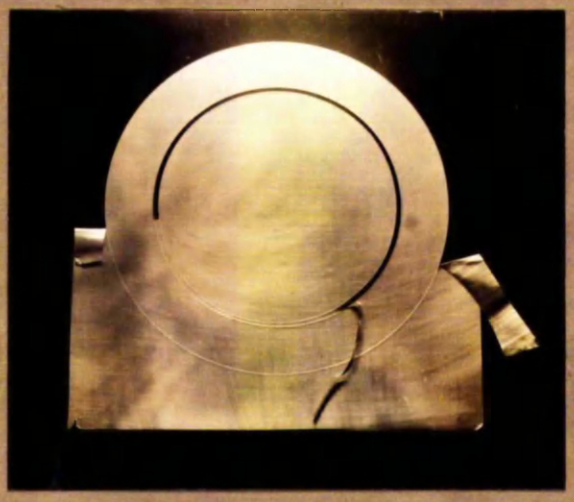


Fig. 8b. Application of tinfoil electrodes.

Flat-Plate Electrodes.

The circular brass plate electrodes used in these tests were constructed as shown in the section given in Fig.9(a). The brass plates were first attached to circular bakelite discs by means of screws through the bakelite engaging in holes tapped into the brass so that the "contact" faces of the brass were not broken. The brass surfaces were then turned and ground flat, the guard ring being obtained by cutting a circular gap as shown. The overall diameter of these electrodes was 15.2 cm.

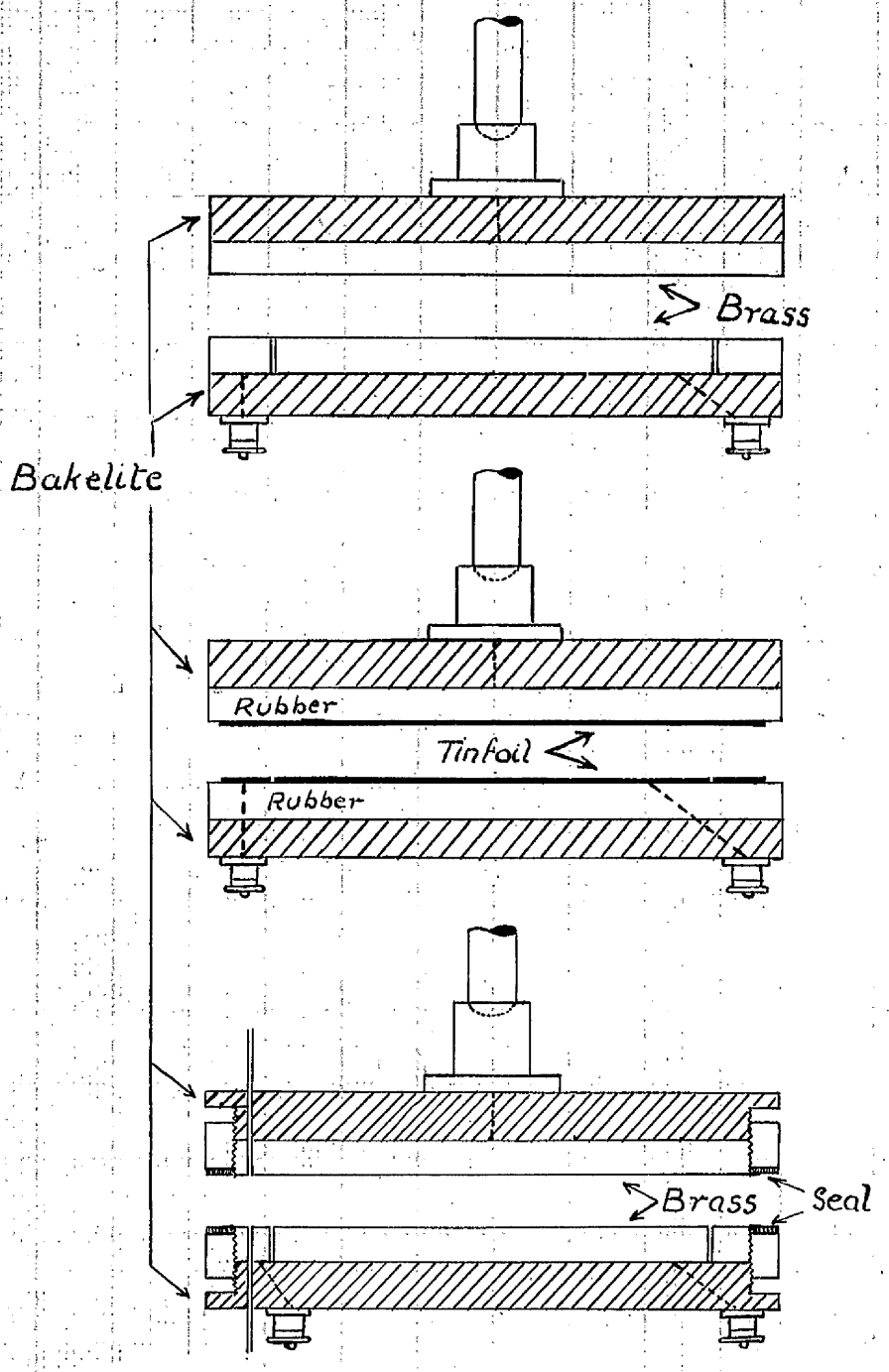


Fig.9

DETAILS OF ELECTRODES

Tin-foil Electrodes.

Tin-foil as an electrode material was employed in several ways. The use of tinfoil in one of the ways described below (i.e. the rubber - tinfoil electrodes) was suggested by consideration of flexibility for the case of sheets which were not perfectly flat, and on which it was obviously impossible to expect good contact with rigid electrodes.

Rubber - tinfoil electrodes were constructed as shown in the section given in Fig.9(b). The backing of a disc of bakelite, as in the case of the flat-plate electrodes, was used, and a circular sheet of rubber attached thereto by means of rubber solution. It was found advantageous to roughen the surface of the bakelite to ensure that the rubber adhered firmly. The tinfoil was attached to the rubber surface with a mixture of petrol and rubber solution. The degree of flexibility permissible in the rubber was found to be limited by the tendency of the rubber to spread under pressure, with consequent distortion of the tin-foil. Contacts indicated by dotted lines were made from the tin-foil to terminals on the bakelite, by means of fine wire.

Another form of tinfoil electrode used was the "transfer" type. Here, circular discs of tinfoil were applied to the surface of the dielectric using a trace of transformer oil as an adhesive. The method first tried was to apply the

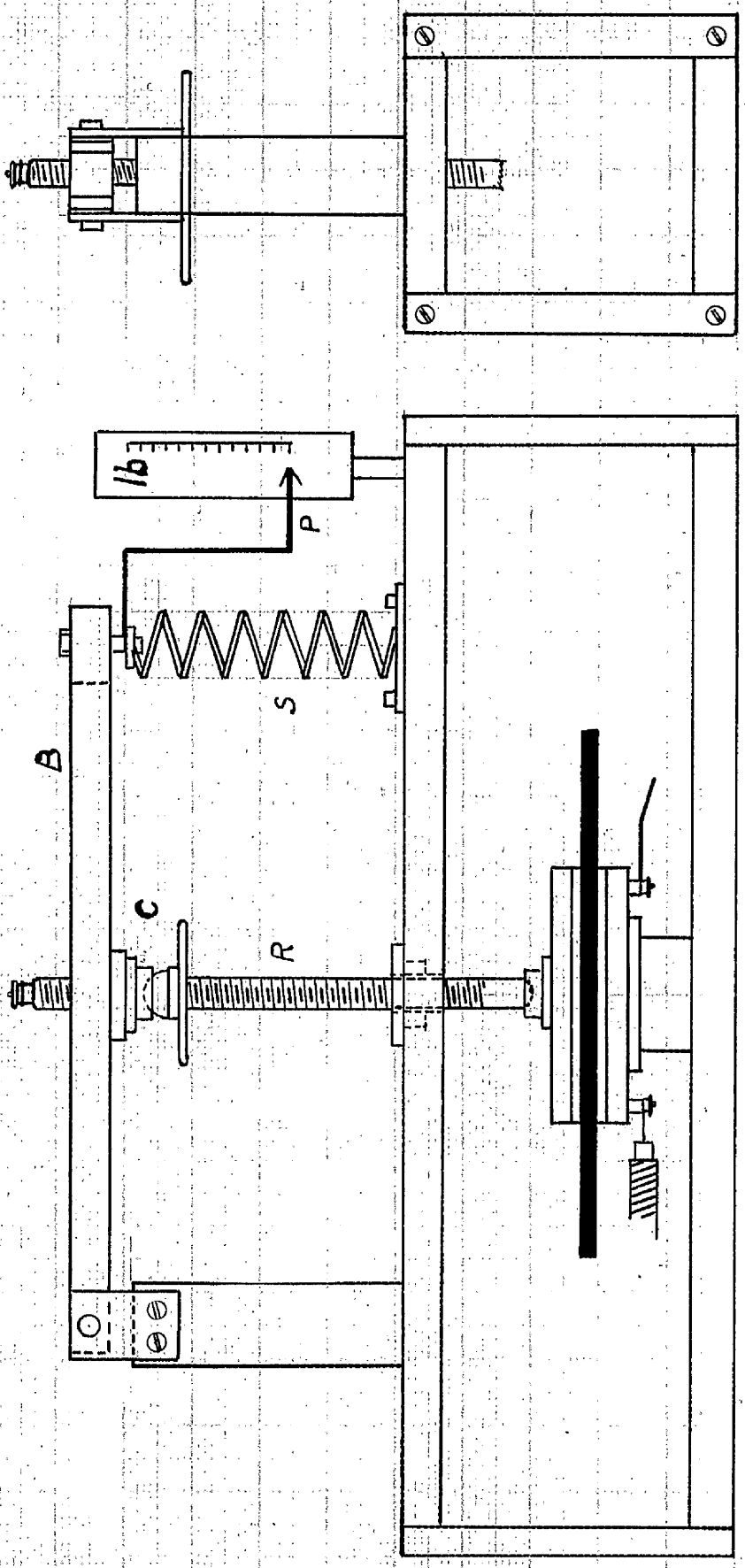
electrodes as irregularly shaped pieces of tinfoil, and, by means of a compass resting on a piece of rubber to prevent damage to the tin-foil, to mark out the shape of the electrodes and the inner and outer diameters of the guard ring gap. The tinfoil was then cut with a sharp knife along the marks left by the compass. A quicker and more certain method of cutting the tin-foil, subsequently used, was to employ a compass, the outer leg of which carried a fine point. After a certain amount of practice, the tin-foil could be cut cleanly by means of the compass point, as shewn in Fig. 8(b). Attempts to devise a compass carrying a double blade, to cut the inner and outer edges of the guard ring gap simultaneously, proved unsuccessful. It was found that the amount of oil necessary to obtain correct adhesion of the tin-foil was rather critical. The oil was applied copiously to the specimen and then wiped off until only a film remained. The tinfoil was then applied and by means of a circular rubbing motion of a cloth, starting at the centre and moving outwards, the tinfoil was pressed into intimate contact with the specimen. Too much or too little oil resulted in the tinfoil moving easily under the rubbing action.

A variation of the above method, which was also used satisfactorily, was to attach the tinfoil electrodes, after cutting, to sheets of flexible material and apply these

sheets, as above, to the specimen. It was found possible, with care, to apply and remove these electrodes to several specimens in turn, whereas with the arrangement used above, new electrodes were necessary for each separate specimen of dielectric. Some difficulty was met with in obtaining a suitable material on which to mount the tin-foil. One method which was used with some success was to attach the tinfoil by means of rubber solution to tracing cloth which had previously been washed in boiling water and ironed flat. Contact with the electrodes when complete was made with strips of tin-foil passing through the cloth and in contact with the tin-foil electrode.

Electrode Holder.

For those tests in which it was desired to apply a variable pressure to the electrode, the arrangement shown in Fig.10 was constructed. This consisted of an upper and lower platform of teak joined at the corners by bakelite strips. A load was applied to the specimen by means of the brass rod *R*, which also served as the high-voltage lead to the upper electrode. Control of the loading was obtained by means of the handwheel *W*, on the screwed rod, the thrust on the insulated beam *B* being taken by a spherical seating between the handwheel and the brass cup *C* attached to the beam. This device allowed for the angularity of the beam when the spring *S* was extended. The extension of the spring was indicated by the pointer *P*, and the scale was calibrated to read directly the load applied to the electrode system.



ELECTRODE HOLDER

Fig.10

EXPERIMENTAL RESULTS.

Four thicknesses of varnish-paper board were used for these tests: 128 mils, 255 mils, 386 mils, 508 mils.

Four types of electrodes were employed: flat plates, rubber-tinfoil, tinfoil-transfer, and aquadag, constructed and applied as described above.

With both flat plate and rubber-tinfoil electrodes, readings were taken over a range of loads, applied to the electrodes by means of the holder described above, from 5 lb to 50 lb.

Determinations of power factor and capacitance were made for each of the above conditions at stresses of 25, 35 and 50 volts per mil.

The values of permittivity given in the succeeding tables and graphs were based on the measured thickness of the specimen.

The results for aquadag and transfer electrodes are compared in Table E.

The results for flat plate, and rubber-tinfoil electrodes are shewn in Figs.11-18, the former by circles and the latter by crosses. For purposes of comparison, the values of power factor and permittivity at each voltage stress, obtained by the use of aquadag electrodes, are indicated by dotted lines.

TABLE E

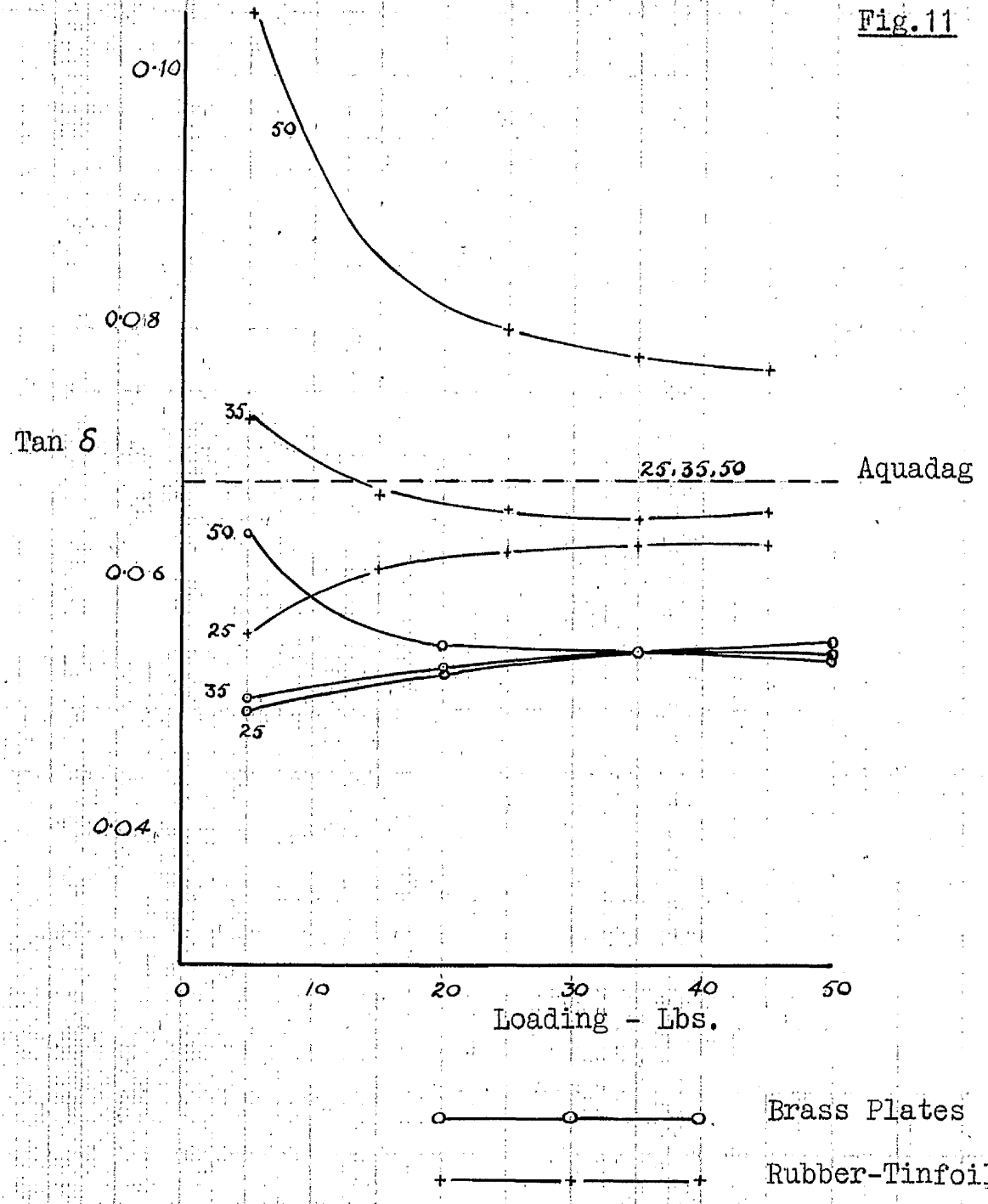
Power Factor and Permittivity of Varnished Paper Boards with Aquadag and Transfer Electrodes.

Thickness of sheet in mils	Stress in volts per mil	Power factor and permittivity	Aquadag	Tinfoil	Percentage change on aquadag values
128	25 35 50	P.F.	0.068 0.068 0.068	0.070 0.072 0.072	2.9 5.9 5.9
255	25 35 50	P.F.	0.041 0.0415 0.042	0.042 0.042 0.042	2.4 1.2 0
386	25 35 50	P.F.	0.0382 0.040 0.041	0.0385 0.039 0.041	0.8 2.5 0
508	25 35 50	P.F.	0.083 0.085 0.0866	0.0828 0.084 0.0861	0.26 1.3 0.58
128	25 35 50	κ	6.26 6.26 6.26	6.15 6.15 6.15	1.8 1.8 1.8
255	25 35 50	κ	5.70 5.70 5.70	5.79 5.79 5.79	1.6 1.6 1.6
386	25 35 50	κ	5.64 5.64 5.65	5.68 5.68 5.68	0.7 0.7 0.5
508	25 35 50	κ	6.62 6.63 6.64	6.41 6.41 6.42	3.1 3.3 3.5

VARNISH PAPER BOARD

Thickness 128 mils

Fig.11

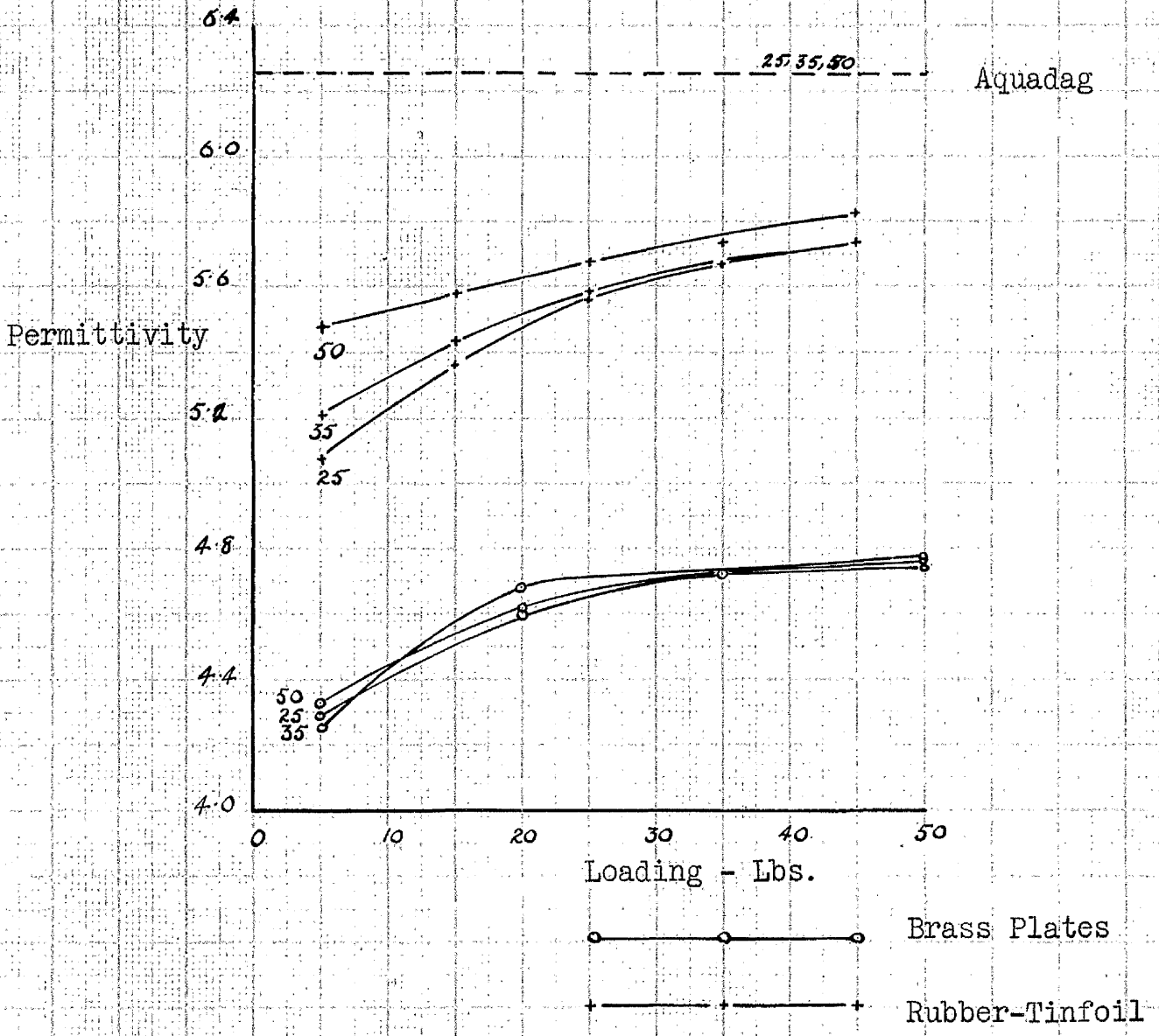


Figures on curves indicate stress in volts per mil.

VARNISH PAPER BOARD

Thickness 128 mils

Fig.12

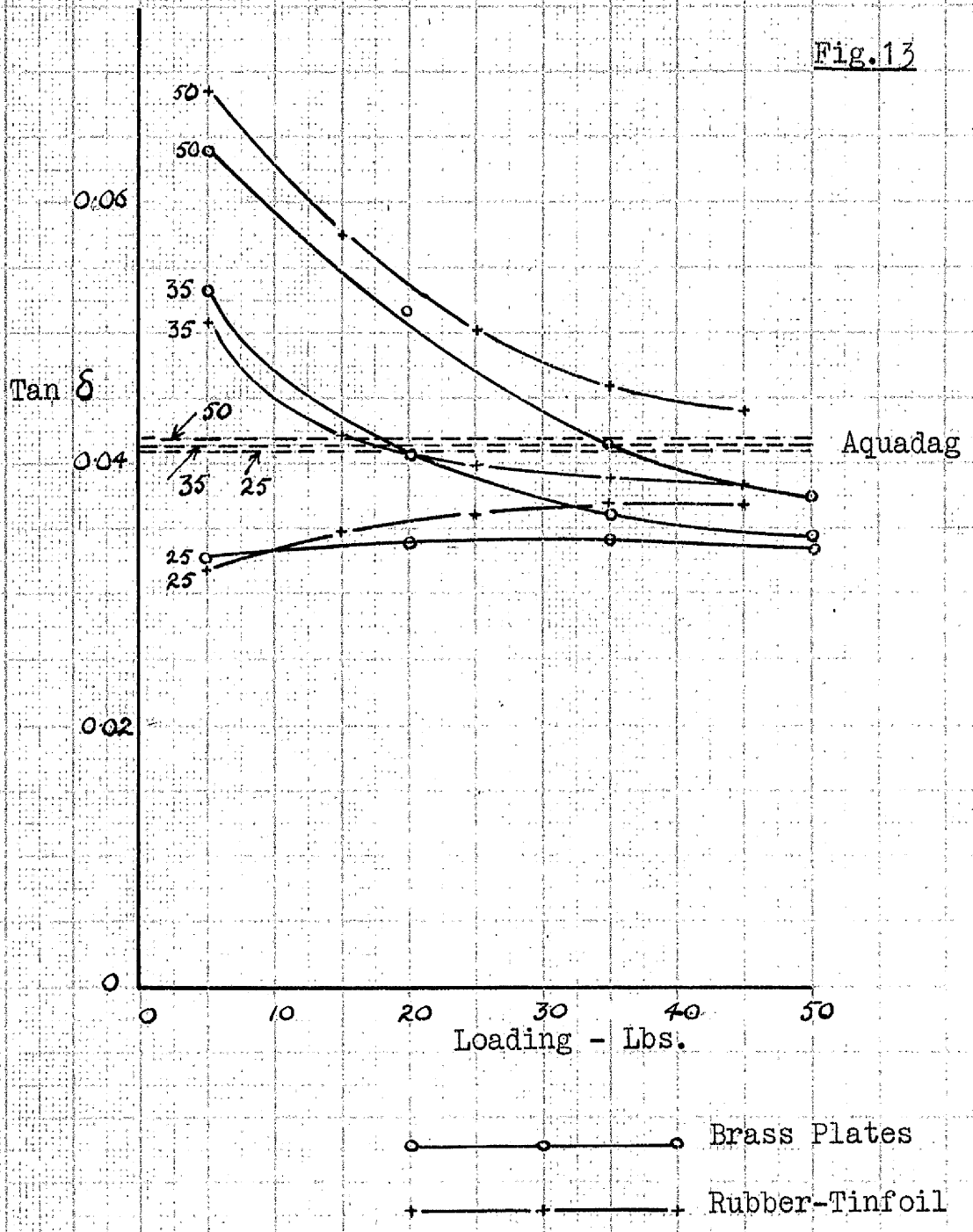


Figures on curves indicate stress in volts per mil.

VARNISH PAPER BOARD

Thickness 255 mils

Fig.13

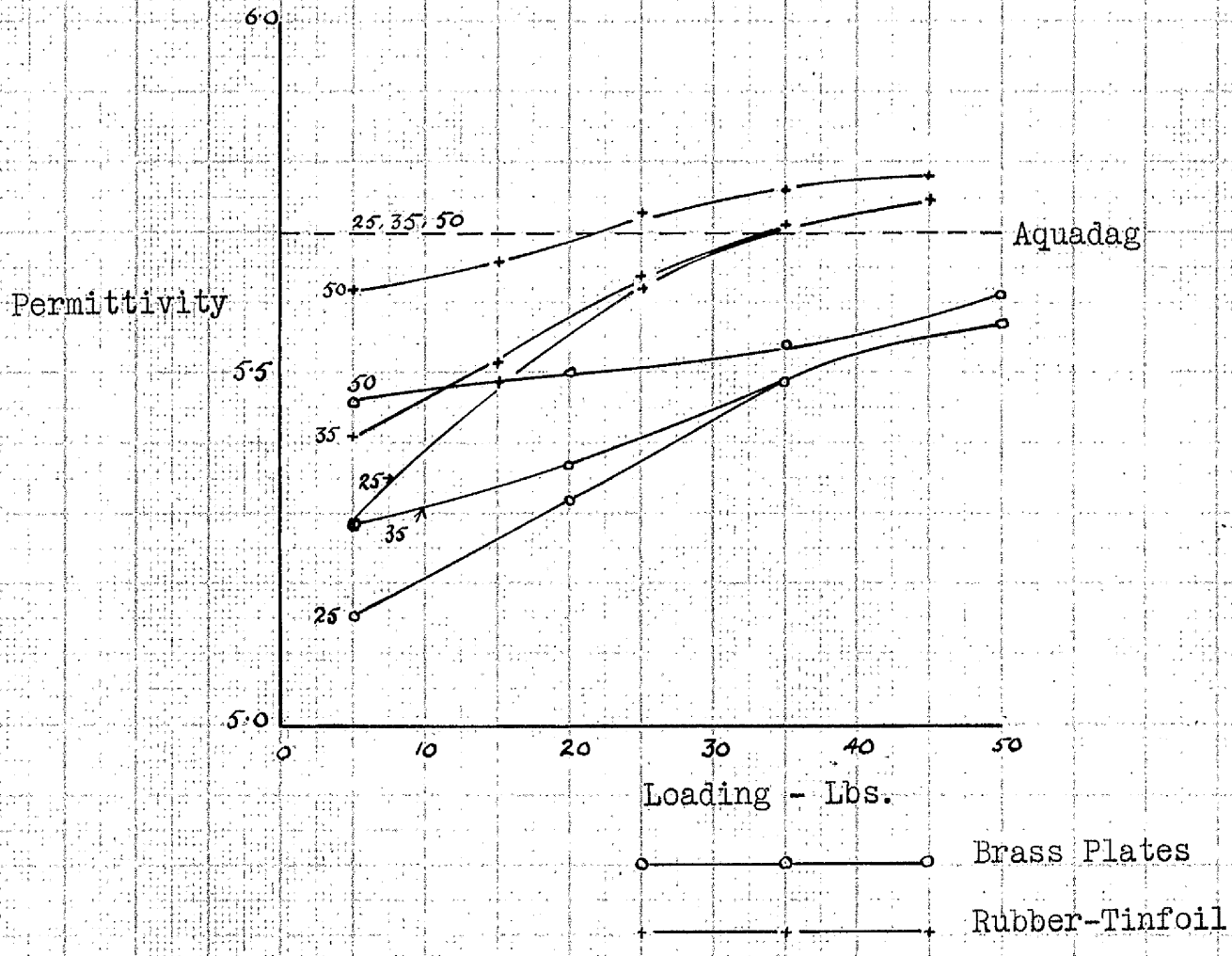


Figures on curves indicate stress in volts per mil.

VARNISH PAPER BOARD

Thickness 255 mils

Fig.14

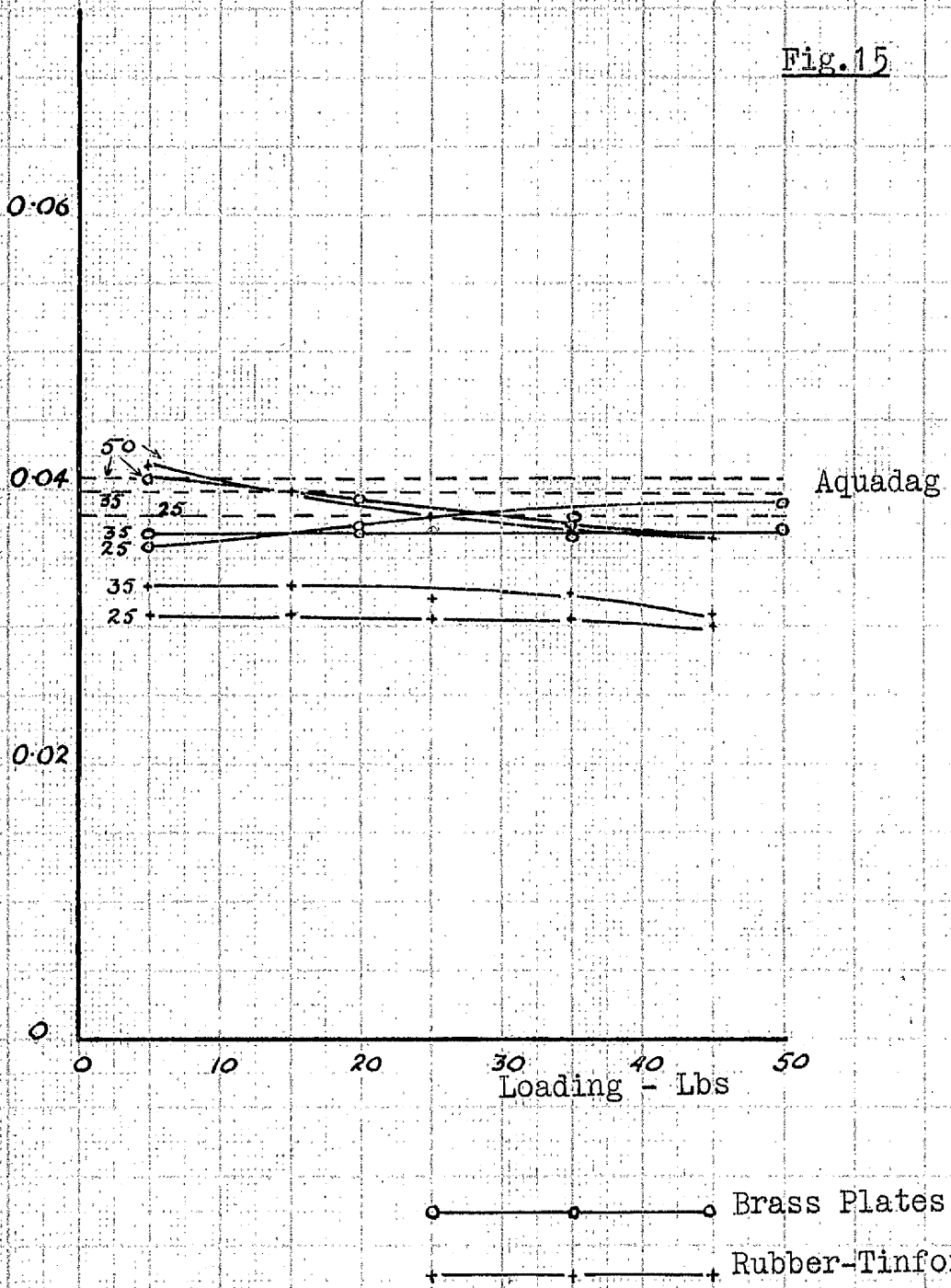


Figures on curves indicate stress in volts per mil

VARNISH PAPER BOARD

Thickness 386 mils

Fig.15

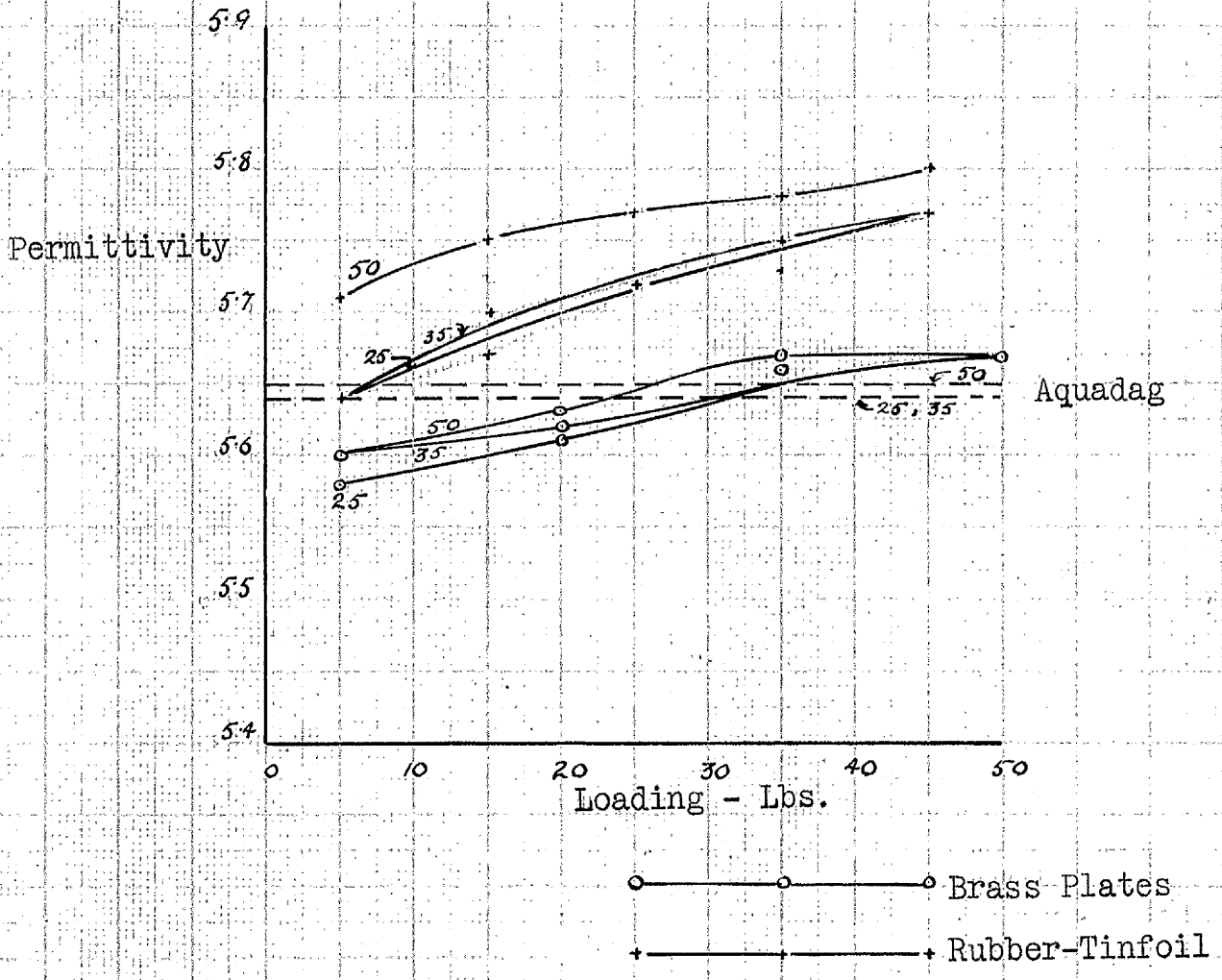


Figures on curves indicate stress in volts per mil.

VARNISH PAPER BOARD

Thickness 386 mils

Fig.16

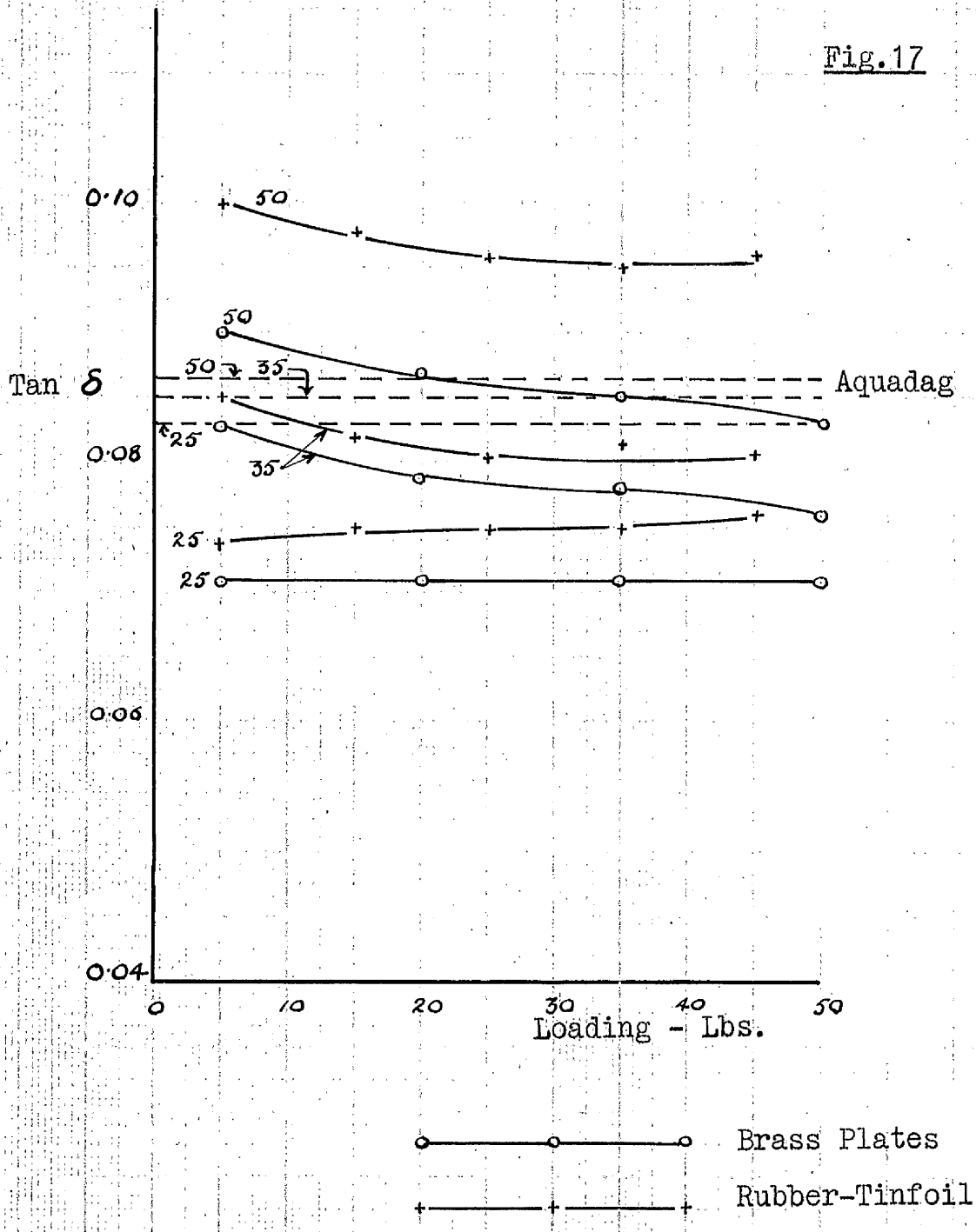


Figures on curves indicate stress in volts per mil

VARNISH PAPER BOARD

Thickness 508 mils

Fig.17

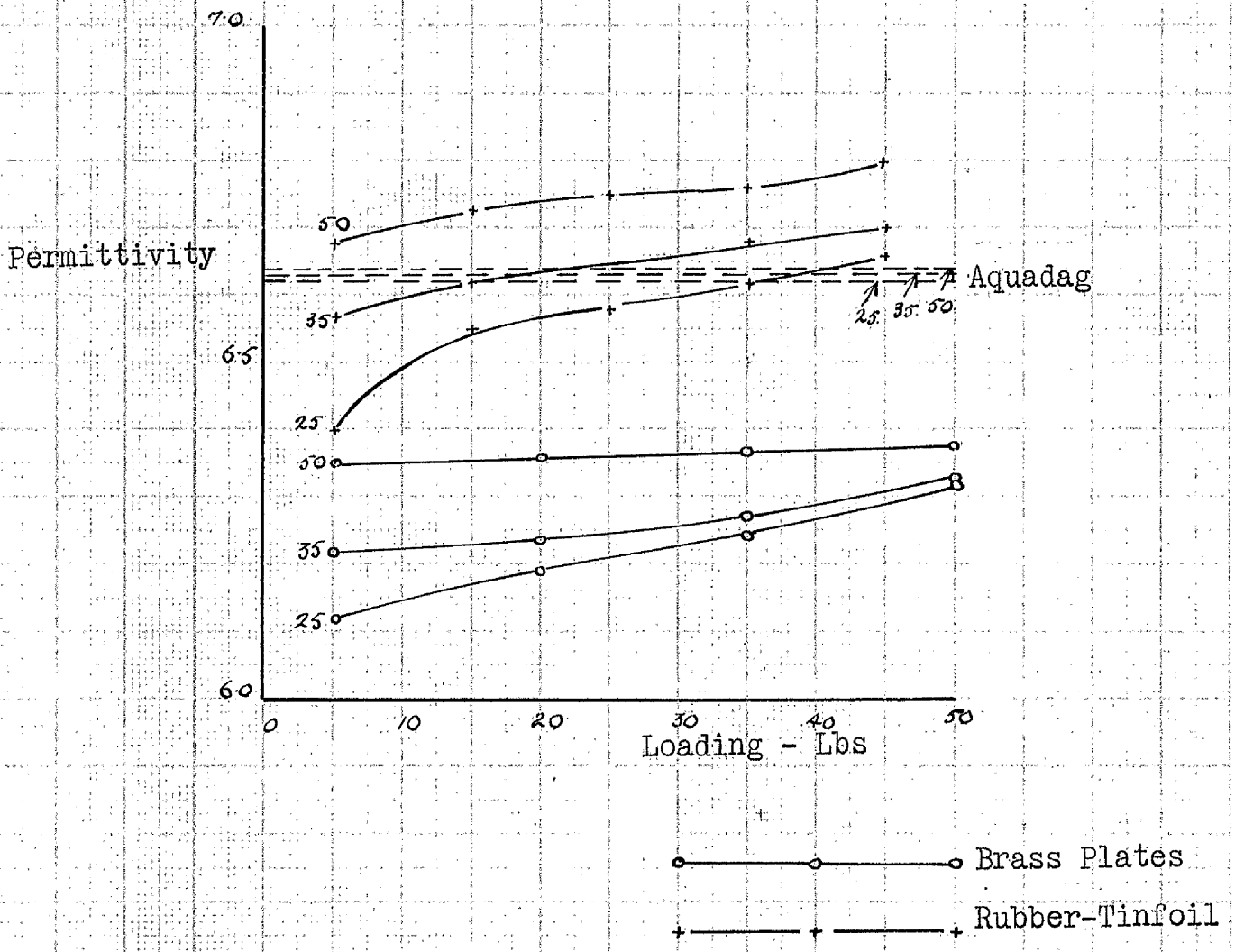


Figures on curves indicate stress in volts per mil.

VARNISH PAPER BOARD

Thickness 508 mils

Fig.18



Figures on curves indicate stress in volts per mil

CONSIDERATION OF RESULTS.

In considering the results, a distinction must be drawn between the behaviour of tinfoil as an electrode when used in conjunction with a rubber backing, and when applied direct, as a transfer.

Tinfoil applied direct.

In this case, the value of the measured power factor is seen not to depend appreciably on the voltage stress, the changes being of the same order as those obtained by the use of "aquadag". These may be due to the power factor - voltage characteristic of the dielectric itself. It is seen that this method offers a very satisfactory order of accuracy.

Owing to the nature of the test, in which each thickness of material has necessarily a different power factor, care must be taken in drawing conclusions as to the effect of thickness on the value of the measured power factor, using these electrodes. It would seem however, that, particularly at low voltage stresses, the influence of the contact effect is most pronounced with thin specimens. This is in accordance with the theoretical considerations outlined on p.19. Equation (ii) can be written

$$\tan\delta = \frac{\tan\delta_D}{1 + C_D/C_c} + \frac{\tan\delta_c}{1 + C_c/C_D}$$

and since the magnitude of C_c is many times that of C_D,

$$\tan\delta \doteq \tan\delta_D\left(1-\frac{C_D}{C_c}\right)-\frac{C_D}{C_c}\tan\delta_c$$

$$\doteq \tan\delta_D-\frac{C_D}{C_c}\left(\tan\delta_D-\tan\delta_c\right) \qquad \text{.....(xv)}$$

Thus, if constant values are assumed for $\tan\delta_D$, $\tan\delta_c$, and C_c, the error in $\tan\delta$ will be proportional to C_D, i.e., inversely proportional to the thickness of the specimen.

The degree of accuracy obtainable by this method is distinctly greater than that usually considered necessary* for industrial measurements, and as it can be relatively easily applied, it is a more satisfactory method than that of using flat plate electrodes for such measurements.

Such variations of measured power factor as do occur, even though they are of the same order as, or in some cases less than those occurring with "aquadag" may be due in part to the behaviour, under varying voltage stresses, of the thin film of oil used for attaching the tinfoil to the specimen. The question of oil films is considered in more detail later.

Flat-Plate and Rubber-Tinfoil Electrodes.

The measurements made with these electrodes are summarised in Figs. 11-18. The curves show that the rubber-tinfoil and flat-plate electrodes exhibit the same general shapes in the

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* See Reference (4)

power factor / loading curves.

The following salient points emerge from an examination of the results:-

(a) The variations in the apparent power factor ($\tan\delta$) are in all cases greater than those in the apparent permittivity (χ)

(b) The variations in $\tan\delta$ with different voltage stresses are in all cases less at high values of loading of the electrodes than at low values.

(c) These variations are most pronounced with the thinner specimens.

(d) With both flat plates and rubber-tinfoil, the effect of increased loading is to reduce $\tan\delta$ in case of high voltage stresses and to increase it in the case of low voltage stresses.

(e) χ increases, in all cases, with increase of the loading.

(f) The values of χ measured with rubber-tinfoil electrodes are always greater than those measured with flat-plates.

(g) At a loading of 45 lb., the values of χ measured with rubber-tinfoil electrodes are, in every case except one, nearer the true values than those obtained with the flat plates, being within about 2% of the aquadag value in the case of the

three thickest specimens and about 7% in the case of the thinnest specimen.

(h) The variations in $\tan\delta$ using flat plates are seen in general to be greater than those reported by Churcher, Dannatt and Dalgleish*, for similar tests, although they also give instances+ where the variation is as great as 0.0222 at stress of 5 volts per mil to 0.0512 at a stress of 45 volts per mil.

The complexity of the results observed with these two types of electrodes is largely due to the presence of an air film of indefinite dimensions between the surface of the dielectric and the electrodes. The film will have properties dependent on the degree of ionization attained, which in turn is dependent on the applied loading, as determining the dimensions of the film and its applied voltage. The properties of such films are considered in greater detail later, in connection with a later series of experimental results. It suffices to say for the moment that the effect of an ionised film in series with the dielectric will, in general, increase the power factor of the combination. This is seen in the present case when, at a given applied pressure, the power factor shows an increase with increased voltage.

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* See Reference (14)
\+ Ibid, infra p.256.

The change of $\tan \delta$ with varying loading at high and low voltage stresses may, it is suggested, be explained on the basis of the above theory by supposing that at low stresses ionization is virtually absent, and the film acts as a pure capacitor whose capacitance is increased as the dimensions of the film are decreased by pressure, thus raising the power factor of the combination. (i.e. $\tan \delta$ is increased in equation (xv) by increase of C_c). The commencement of ionization at smaller film thicknesses would also tend to increase the power factor.

At high voltage stresses the action is not so clear but if ionization is considered to be well advanced, reducing the film thickness will have the effect of reducing the volume of ionized air, and therefore the loss.

It is seen that both the flat plate and rubber-tinfoil type of electrodes are unsuitable for anything more than rough approximations of power factor. If they are used, as large an applied load as is convenient should be used to ensure consistent results. The rubber-tinfoil type appears to be definitely superior to the flat plate type for the thinner specimens, but this superiority is not maintained in the case of the thicker specimens. The result may be due to considerations of curvature and rigidity of the specimens.

It may be noted that for a given curvature of a specimen the error introduced into the power factor measurement by the air film so formed will be dependent on the diameter of the electrodes, since the maximum thickness of the air film is proportional to the square of the diameter of the electrode, for small curvatures of the specimen.

B. FLAT PLATE ELECTRODES WITH CONTROLLED AIR FILMS.

The complexity of the results recorded in the previous section is due largely to the behaviour of the indeterminate air films between the electrodes and the dielectric. The measurements in the present section with air films of definite dimensions were undertaken in an attempt to obtain a clearer conception of the behaviour of these films, and also as a preliminary to the investigation, in the following section, of the use of controlled liquid films between electrodes and dielectrics.

APPARATUS AND METHOD.

To reduce the number of variables, all measurements were made on a sample of glass, 265 mils thick. Glass was chosen because of the much greater constancy of its electrical characteristics as compared with synthetic-resin varnish-paper boards. (The characteristics of the latter are found to be dependent on the conditions under which they are stored and on the time which has elapsed since they were stoved).

The flat-plate electrodes used were identical with those described on p.31. The air films of various thicknesses were obtained by the insertion, at the edges of the electrodes, of small mica spacers of the requisite thickness. In all the tests a constant load of 10 lbs. was applied to the upper (h.v.) electrode, by means of the apparatus described on p.36, and in each test, measurements of apparent power factor and permittivity were made over a range of nominal voltage stresses up to 75 V per mil. The conditions of the tests were as follows:

1. Normal flat-plate electrodes with
   (a) gaps of nominal values 0, 1.5, 2.5, 5 and 11 mils on the l.v. side only.
   (b) gaps of nominal values 0, 1.5, 2.5, 5 and 11 mils on the h.v. side only.

2. (a) aquadag h.v. electrode and flat-plate l.v. electrode. Gaps of nominal values 0, 3 and 5 mils on the l.v. side.

   (b) aquadag l.v. electrode and flat-plate h.v. electrode. Gaps of nominal values 0, 3 and 5 mils on the h.v. side.

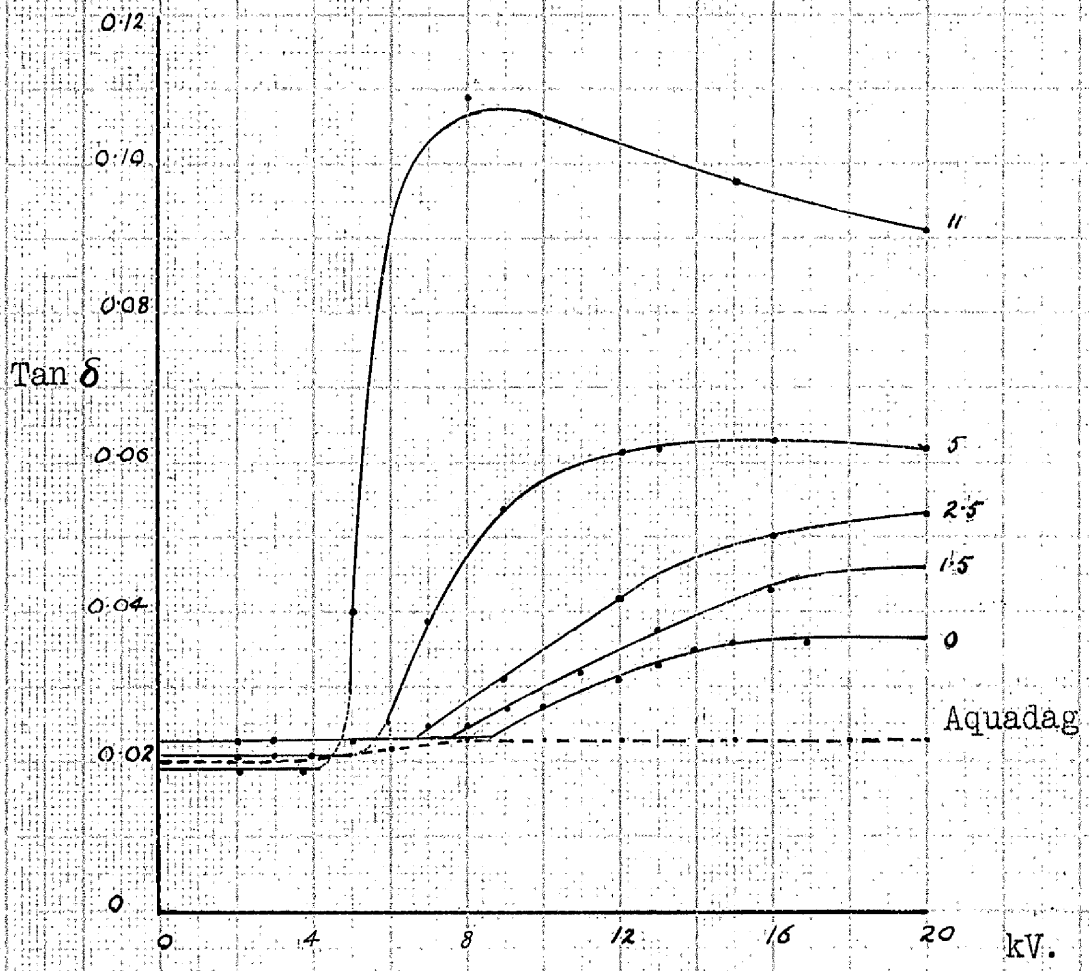
EXPERIMENTAL RESULTS.

The results obtained from the foregoing series of tests are summarised in Figs. 19-26. The values of permittivity, calculated from the measured values of capacitance, are based on a value of t (see p.26) equal to $(265 + t_A)$ mils, where t_A is the nominal thickness of the air film.

On each set of curves, a dotted line indicates the value of $\tan \delta$ or $\varkappa$ measured with aquadag electrodes on both sides of the specimen.

GLASS

Thickness 265 mils



Air gaps on L.V. side. Nominal values, in mils, indicated on curves.

Fig.19

GLASS

Thickness 265 mils

Air gaps on L.V. side. Nominal values, in mils, indicated on curves.

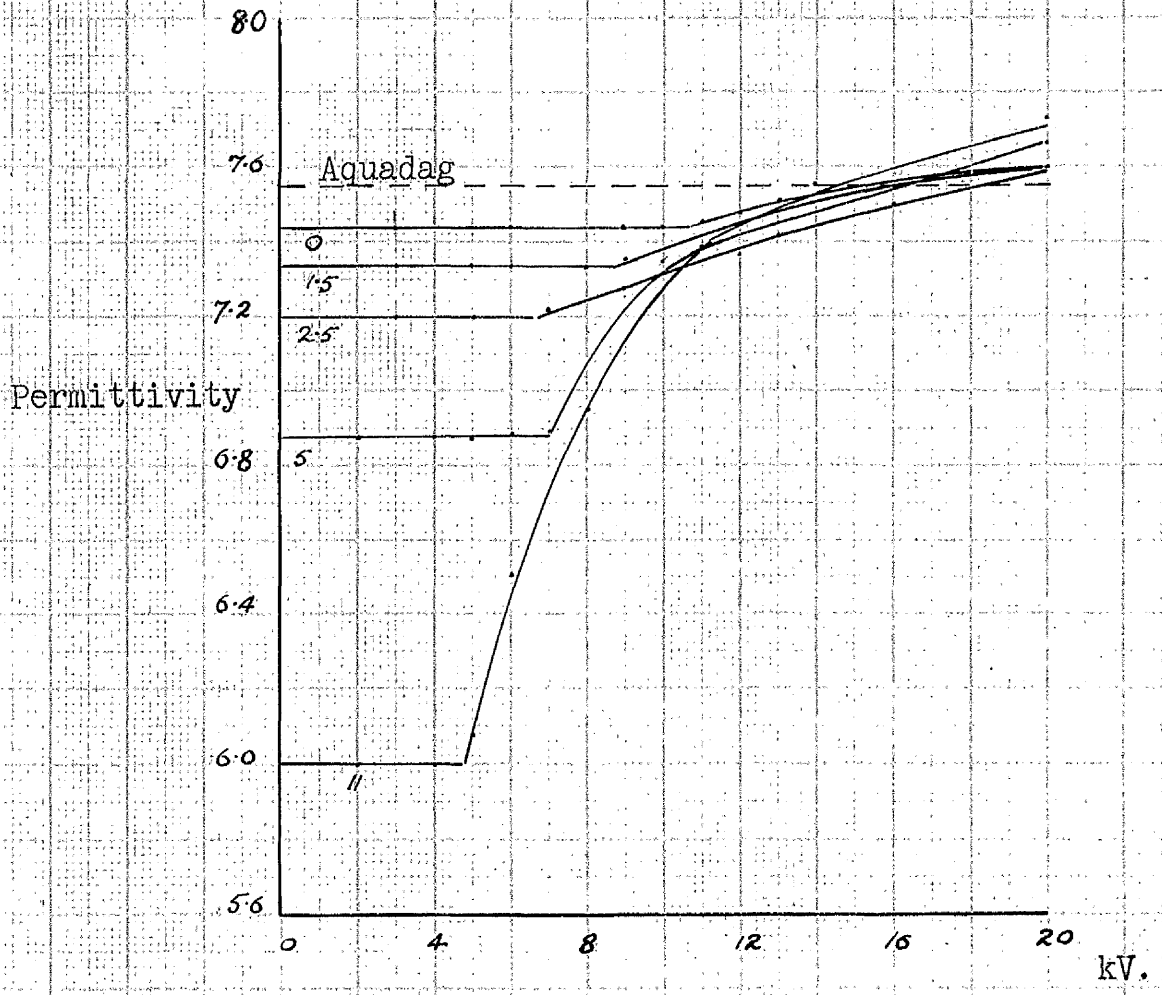
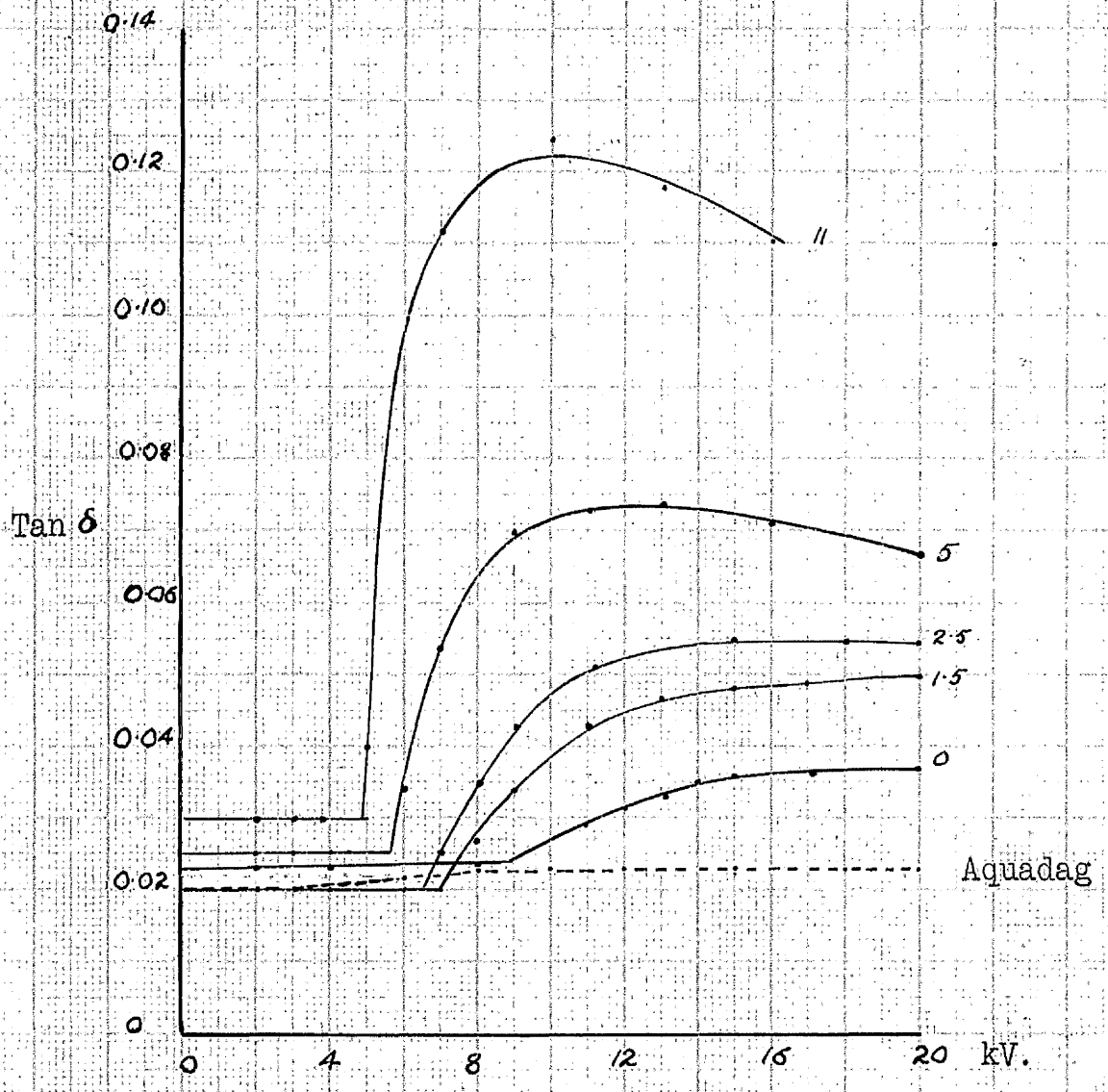


Fig.20

GLASS

Thickness 265 mils



Air gaps on H.V. side. Nominal values, in mils, indicated on curves.

Fig.21

GLASS

Thickness 265 mils

Air gaps on H.V. side. Nominal values, in mils, indicated on curves.

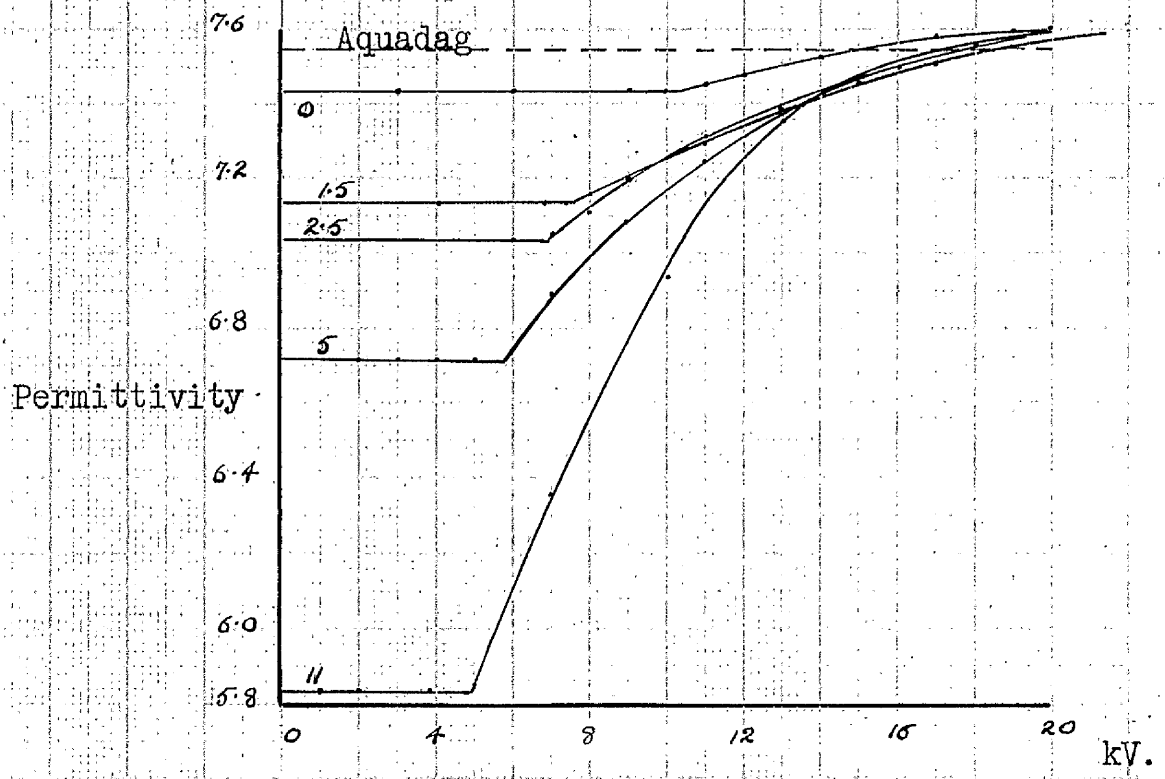


Fig.22

GLASS

Thickness 265 mils

Air gaps on L.V. side. Aquadag on H.V. Nominal values of gaps in mils, indicated on graphs.

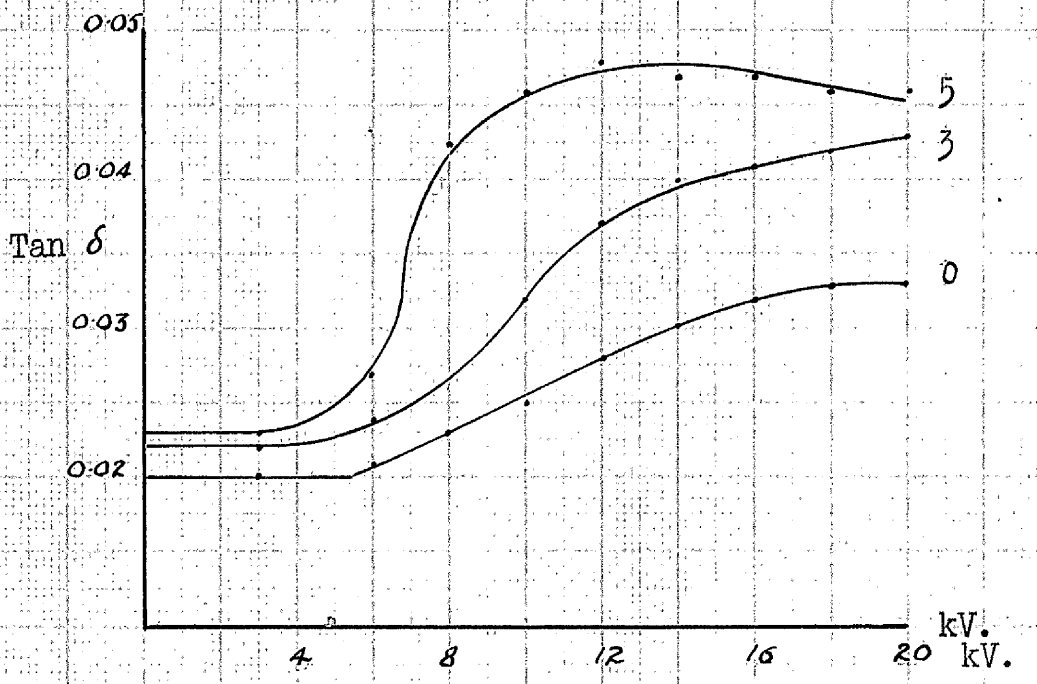


Fig.23

GLASS

Thickness 265 mils

Air gaps on L.V. side. Aquadag on H.V. side. Nominal values of gaps, in mils, indicated on curves.

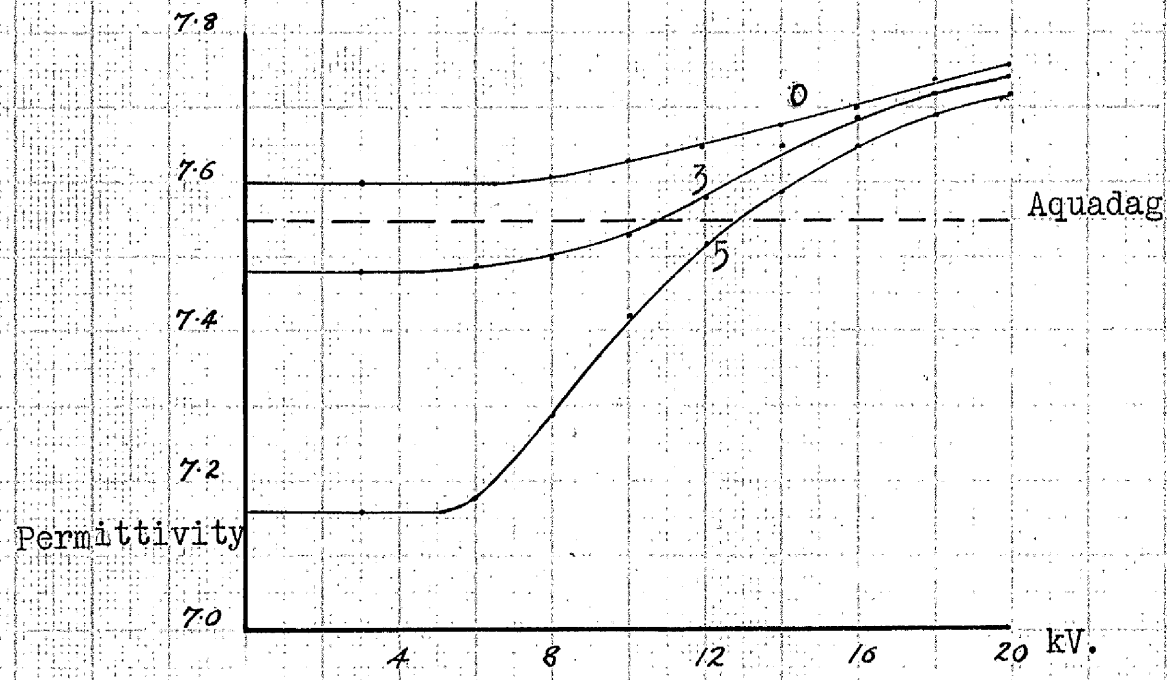


Fig. 24

GLASS

Thickness 265 mils

Air gaps on H.V. side. Aquadag on L.V. Nominal values of gaps, in mils, indicated on curves.

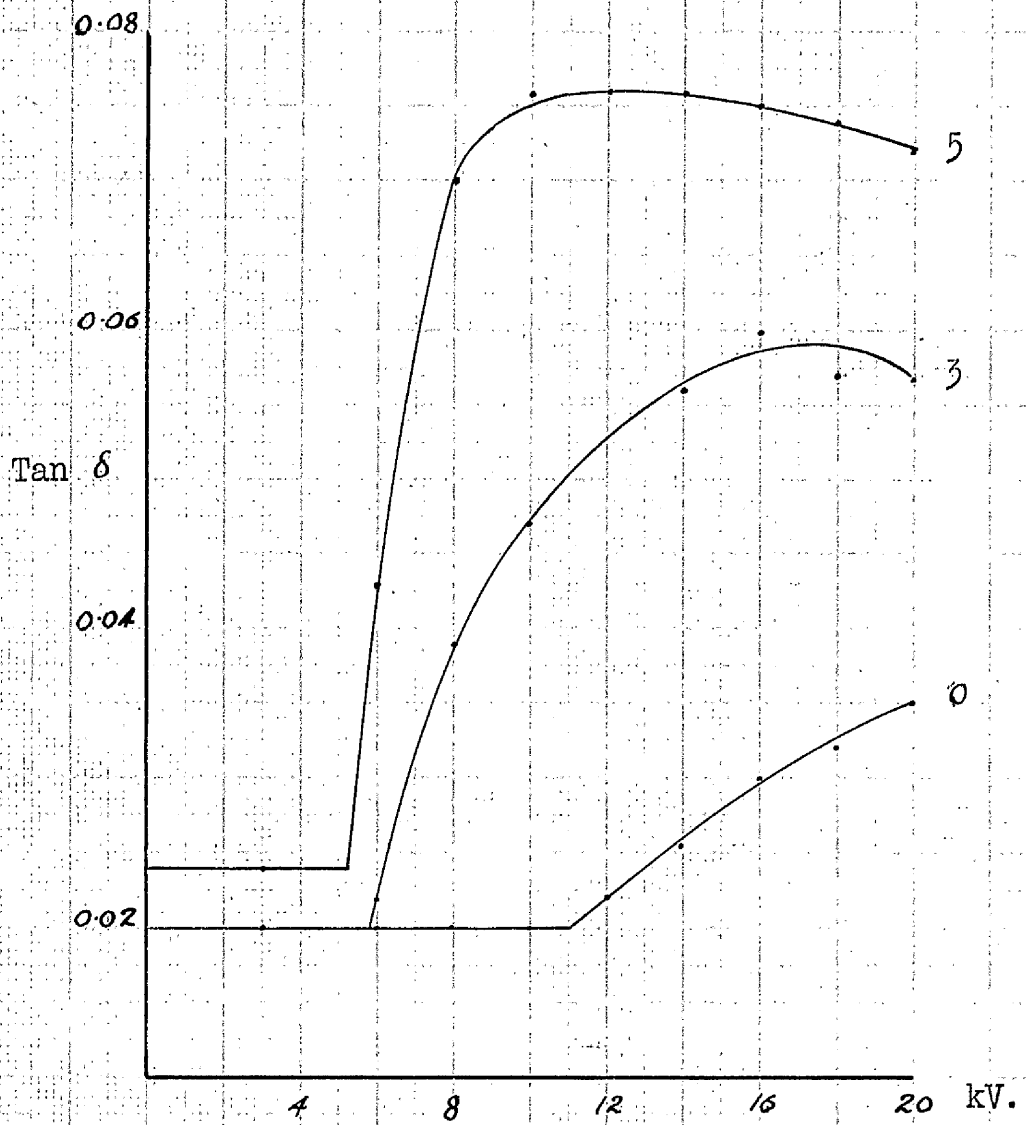


Fig.25

GLASS

Thickness 265 mils

Air gaps on H.V. side. Aquadag on L.V. side. Nominal values of gaps, in mils, indicated on curves.

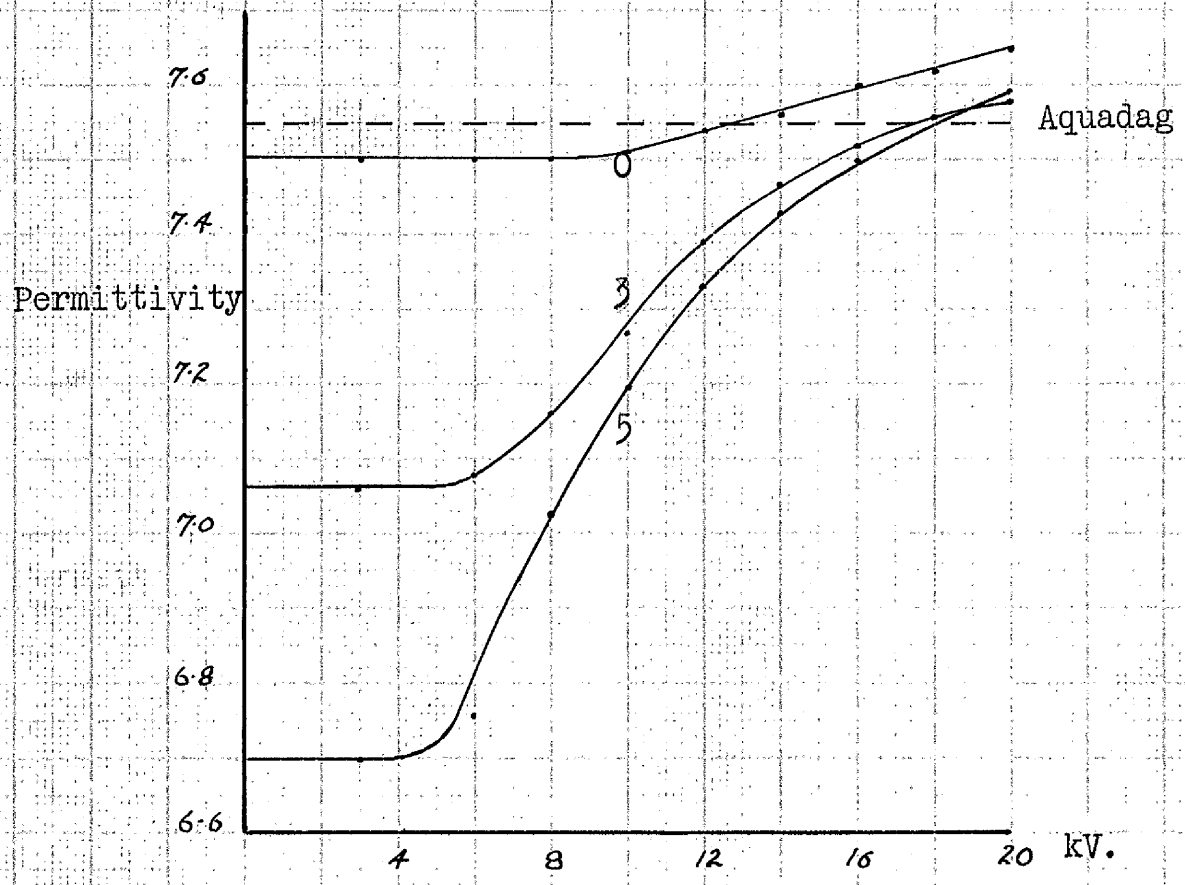


Fig.26

CONSIDERATION OF RESULTS.

The behaviour of a film of air in series with a solid dielectric depends largely on the voltage stress in the film. If the stress is such that ionization has not commenced, the effect of the film on the properties of the composite dielectric can be calculated on the basis of the expressions given on p.19, since, under these conditions, the permittivity $\varkappa_A$ and the loss angle $\tan \delta_A$ of the air film approximate to unity and zero respectively*.

Each of the permittivity/applied voltage and power factor/applied voltage curves in Figs. 19-26 exhibit two salient characteristics.

(a) up to a certain voltage, the values of $\varkappa$ and $\tan \delta$ remain constant.

(b) at a certain critical voltage, the values of $\varkappa$ and $\tan \delta$ commence to increase rapidly with increase of the applied voltage.

It is reasonable to assume that the initial flat portions of each of the curves represent conditions under which the air films are unionised and that the discontinuities in the curves mark the onset of ionization of the films.

---

* Methods for the measurement of power factor and permittivity in which an allowance is made for air films are described by W.J.Sulston in a contribution to the discussion of Fleming & Churcher's paper. See Reference (24).

On these assumptions, the value of κ_D may be found, from (i), in terms of the overall measured permittivity κ, thus

$$\frac{t_A + t_D}{\kappa} = \frac{t_D}{\kappa_D} + \frac{t_A}{\kappa_A}$$

whence

$$\kappa_D = t_D \kappa / (t_A + t_D - t_A \kappa)$$

The values of κ_D so obtained from Figs. 20, 22, 24 and 26 are given in Table F. With the exception of the two results marked * , the calculated values of κ_D are in sufficiently close agreement with the true value, to tend to confirm the assumptions made above. The explanation of the two results so marked, is not apparent. It may be noticed that the value of κ_D , marked +, in the same group of tests is <u>higher</u> than the true value of 7.55, for the case of zero thickness of the air film.

The true permittivity of the dielectric, κ_D , is sensibly constant for a wide range of voltage gradients, as is also $\tan \delta_D$. Were this not the case, the above calculation would be invalidated by the change of voltage gradient in the dielectric due to the interposition of the air films.

The voltage gradients in the air film and in the dielectric, g_A and g_D, can be calculated in terms of the applied voltage V, and thicknesses of the air film and dielectric, t_A and t_D. On the basis of Fig. 4, the ratio of the voltage V_A across the film, to that across the dielectric V_D is

TABLE F.

Reference	t_A, mils	χ observed	χ_D calculated	χ_D By aquadag
Fig. 20	11	6.00	7.58	7.55
"	5	6.88	7.74	7.55
"	0	7.43	7.43	7.55
Fig. 22	11	5.84	7.30	7.55
"	5	6.72	7.52	7.55
"	0	7.44	7.44	7.55
Fig. 24	5	7.16	8.08 *	7.55
"	3	7.48	8.08 *	7.55
"	0	7.60	7.60 +	7.55
Fig. 26	5	6.70	7.51	7.55
"	3	7.06	7.58	7.55
"	0	7.50	7.50	7.55

$$V_A / V_D = t_A K_D / t_D K_A \quad *$$

whence the voltage gradients in the air film and specimen are

$$g_A = V / \left[t_A \left(1 + \frac{t_D K_A}{t_A K_D} \right) \right] \qquad \text{.....(xvi)}$$

and $$g_D = V / \left[t_D \left(1 + \frac{t_A K_D}{t_D K_A} \right) \right].$$

In applying electrodes with an appreciable air gap therefore, the actual voltage stress in the dielectric is appreciably less than the nominal stress V/t_D, the ratio being +

$$1 / \left(1 + \frac{t_A K_D}{t_D} \right) \qquad \text{.....(xvii)}$$

This ratio is plotted for various thicknesses of film and dielectric and for $K_D = 4$ and $K_A = 8$, in Fig. 27.

Although the measured values of K show a reasonable agreement with the calculated values for the composite dielectric this is not the case with $\tan \delta$, the measured power factor of the composite dielectric. The initial

---

* When $\tan \delta_A$ and $\tan \delta_D$ are both small; strictly

$$\frac{V_A}{V_D} = \frac{t_A K_D}{t_D K_A} \left(\frac{1 + \sin^2 \delta_D}{1 + \sin^2 \delta_A} \right)^{\frac{1}{2}}$$

\+ The exact expression for the ratio is

$$1 / \left[\frac{\cos \delta_D K_D}{t_D} \left\{ \left(\frac{t_A}{K_A} \tan \delta_A + \frac{t_D}{K_D} \tan \delta_D \right)^2 + \left(\frac{t_A}{K_A} + \frac{t_D}{K_D} \right)^2 \right\}^{\frac{1}{2}} \right]$$

which reduces to (xvii) when $\tan \delta_A = 0$, $\tan \delta_D = 0$ and $K_A = 1$.

DIELECTRIC AND AIR FILM, NO IONISATION.

Theoretical ratio of stress on dielectric when a voltage V is applied to the combination, to the nominal stress, V/t_D.

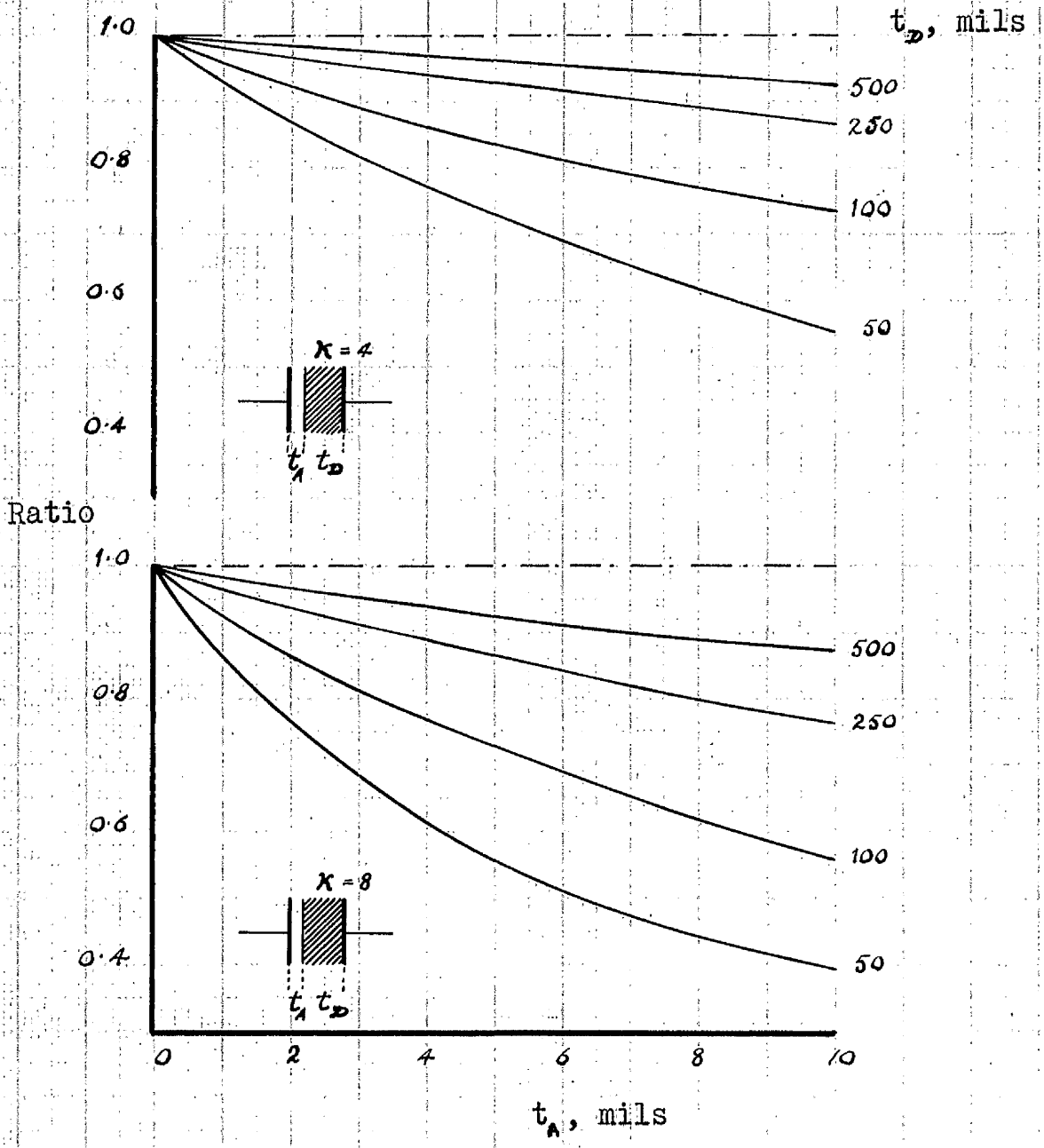


Fig.27

(unionised) values of $\tan\delta$ should, if $\tan\delta_A = 0$ and $K_A = 1$, decrease with the increase of the thickness of the air film. For, from (ii),

$$\tan\delta = \tan\delta_D \frac{1}{1 + \frac{C_D}{C_A}}, \qquad \text{.....(xviii)}$$

whence $\tan\delta$ should decrease with increase of t_A, i.e., with decrease of C_A; actually it is seen that in nearly all cases a slight increase in $\tan\delta$ is observed. Taking for example Fig.23, where $\tan\delta_D = 0.020$ (aquadag value), $t_D = 263$ mils and $K_D = 7.55$, the calculated value of $\tan\delta$ for a film thickness of 5 mils is

$0.020 \Big/ \left(1 + \frac{5 \times 7.55}{265}\right) = 0.018$ as compared with the measured value of 0.023.

No satisfactory explanation can be advanced for this anomaly. It might be thought that the air film, of negligible thickness, on the other side of the specimen in Figs. 19-22, is a factor in the result; Figs. 23-26 however show the same increase of power factor with increase of air film thickness, and in these cases aquadag electrodes are used on the other side of the specimen. It may be noticed that in Figs. 17-19 some values of $\tan\delta$ at low voltage stresses are markedly lower than the true value $\tan\delta_D$. In computing the effect of air films included

between several layers of varnished cloth, Hartshorn* mentions that the experimental and calculated values of power factor differ by amounts which are greater than can be accounted for by experimental errors and suggests that the discrepancies may be due to the conductance of the air film.

The voltage at which the values of $\varkappa$ and $\tan\delta$ commence to increase from their hitherto steady values is seen in nearly every case to be very clearly defined. In each electrode arrangement this critical voltage is lowest for the case when the thickness of the air film is greatest. Thus, in Fig.21, the critical voltages for films of nominal thickness 0, 1.5, 2.5, 5 and 11 mils are 8.6, 7.6, 6.6, 5.2 and 4.2 kV respectively. This variation in the value of the critical voltage with the film thickness, is thought to be related to the variation in the electric strength of air with electrode spacing, at small spacings. Values of the electric strength of air under similar conditions of "restricted ionisation"+ to those of the present tests have been determined by Dubsky‡, and a curve shewing the high values of electric strength at small spacings is shewn in Fig.28.⊕

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* See Reference (12)
+ The term used by Dawes and Hoover (See Ref.20) in connection with the ionisation of cable voids.
‡ See Reference (25)
⊕ Together with the method used; the critical gradient was obtained by visual observation of the beginning of the discharge.

ELECTRIC STRENGTH OF AIR FILMS IN A COMPOSITE DIELECTRIC

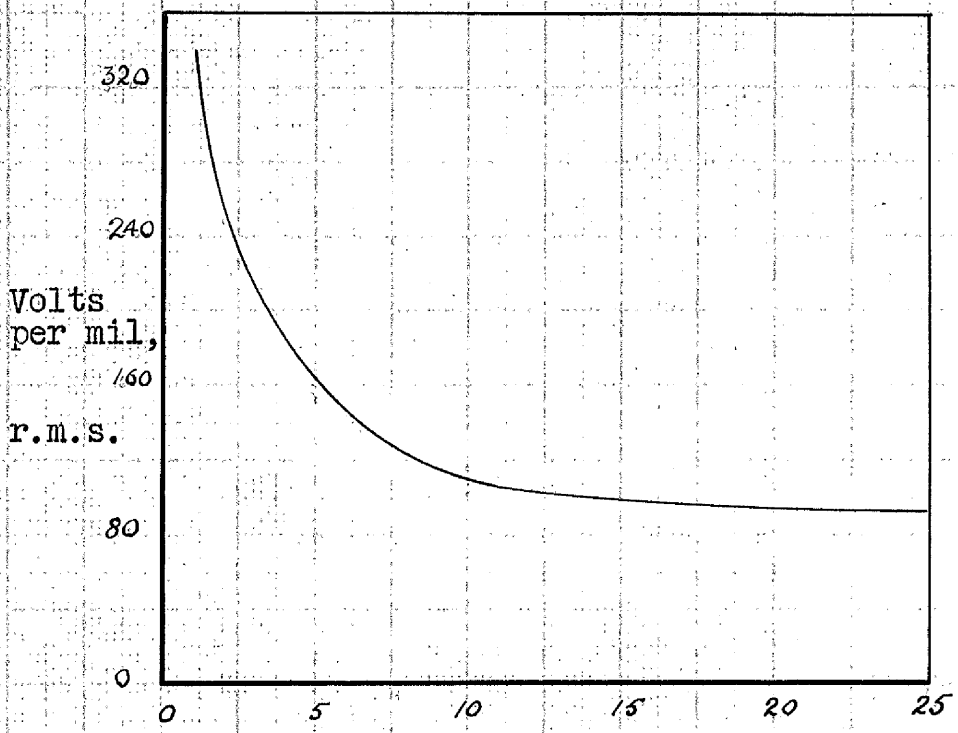
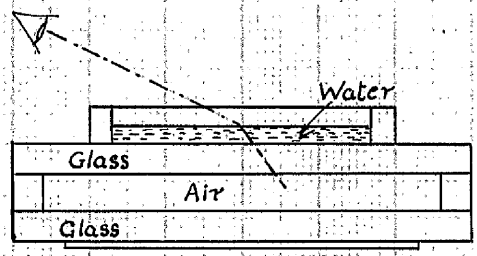


Fig.28



Method of determination of critical gradient

FROM F.Dubsky, Trans. A I E.E.; Vol. 38, p. 537

It is seen from (xvi) that for a given applied voltage, g_A decreases by only a small amount as $t_{A\neq}$ is varied from 0 to 11 mils. The change of t_A has therefore the main effect of varying the electric strength of the film, with a consequent change in the critical voltage at which $\tan\delta$ begins to vary rapidly, the voltage decreasing as the film thickness is increased.

The values of voltage gradient in the air film at the critical voltages, calculated from (xvi) are the same order as those given by Dubsky as shewn in Fig.28. Thus, from Fig.21, taking $\chi = 7.55$ and $t_D = 265$ mils, the values of g_A for the 11, 5, 2.5 and 1.5 mil films are 91, 130, 175 and 210 V per mil. From Figs. 23 and 25, where the uncertain thickness of the air film (nominally of zero thickness) on the opposite side of the specimen does not complicate the problem, the values for the 5 mil films are 100 and 138 V per mil and for the 3 mil film 143 and 165 V per mil.

The characteristics of an electrode assembly involving air films on one or both sides of a specimen are similar to those of a cable having voids in which ionisation occurs at certain voltages. The power factor /voltage characteristics of such cables as described by Dunsheath*, Birnbaum+, Schrader‡ and others (e.g., Fig.29) have a shape similar to

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* See Reference (26)
+ Ibid (27)
‡ Ibid (28)

IONISATION IN CABLES

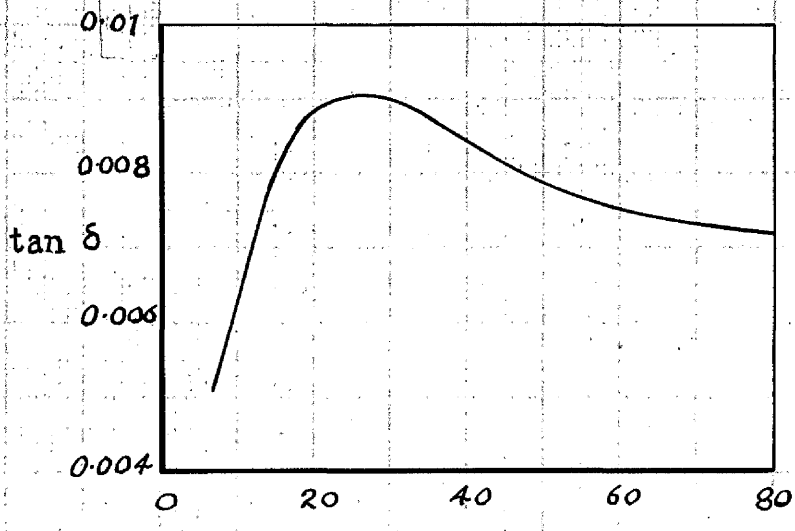


Fig.29

FROM P.Dunsheath, J.I.E.E., Vol. 73, p. 324.

Fig.30a

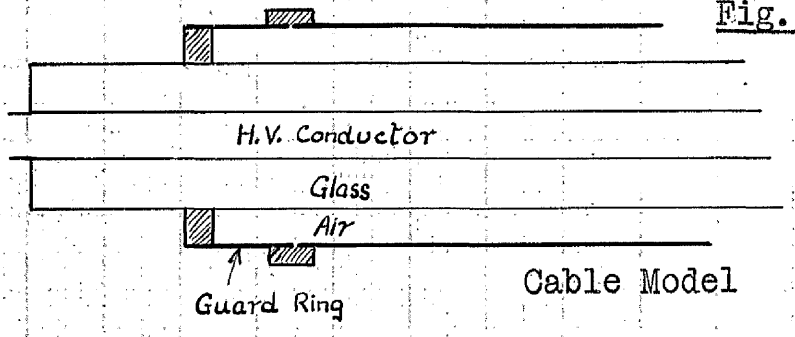
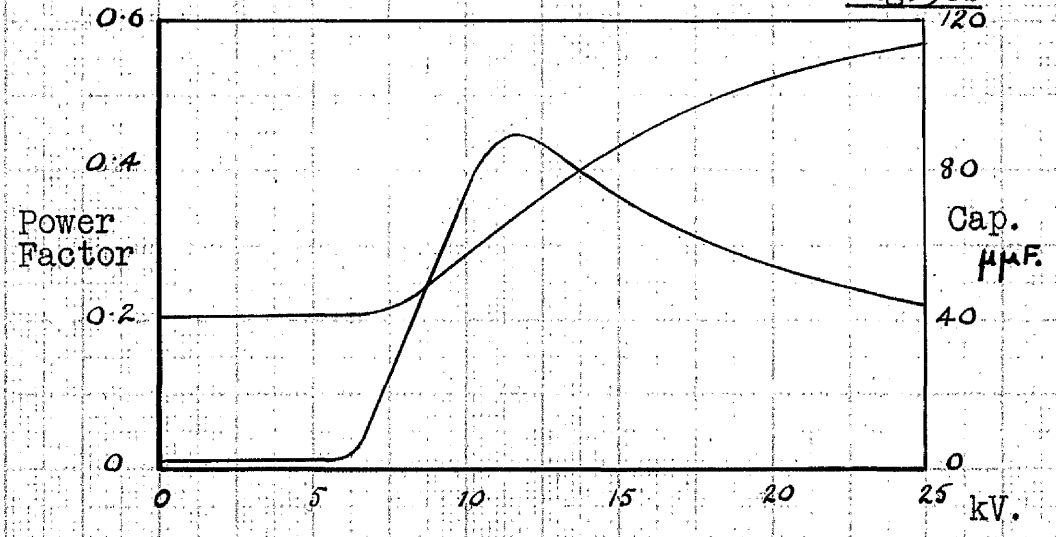


Fig.30b



FROM C.L.Dawes and P.L.Hoover, J.Am. I.E.E., Vol.45, p. 337.

those of the power factor /voltage characteristics of Figs. 19, 21, 23 and 25. The resemblance is further illustrated by the results obtained by Dawes and Hoover* from a model (Fig.30a) to illustrate the ionisation phenomena in a cable. These results (quoted in Fig.30b) illustrate that the power factor is appreciably constant up to a critical voltage, above which it rises to a maximum value and thereafter falls as the voltage is increased. It is seen from this graph that the maximum value of power factor occurs at a voltage equal to twice that at which the power factor begins to increase rapidly. Dawes, Reichard and Humphries+ show analytically that if the loss in a cable is assumed to follow a law of the form

$$P = aV^2 + b(V - V_0)^2$$

where the second term indicates the loss due to ionisation, above a critical voltage V_0, then the maximum value of power factor occurs at a voltage $V_m = 2V_0$. This relation is observed in the cases of the 11 mil air film in Figs.21 and 23. For thinner films the voltage at which the power factor reaches a maximum is in general rather more than twice the critical voltage, a result which may perhaps be due to the uncertainty of alignment of the electrode surface and dielectric at small film thicknesses.

The portions of the curves at voltages above the

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* See Reference (20)
\+ Ibid. (29)

critical voltage are of no practical value* as far as the measurement of $\tan\delta$ and κ are concerned. The results do however serve to illustrate the fact that, if flat-plate electrodes are used, the applied voltage must be such that the critical gradient in the film is not reached. Increase of the film thickness is a disadvantage if it is desired to test the dielectric at high voltage stresses, as the critical voltage is thereby reduced, as shewn above. It will be noticed that, in Fig.19, for example, the critical voltage of 9 kV for the "zero" thickness of film corresponds to an overall gradient of 34 V (r.m.s.) per mil. 50 V per mil is a gradient at which measurements are often made and it is seen that in this case such a gradient would be above that at which the measured power factor is unstable. In practice, it is possible to judge whether measurements are being made below or above the critical voltage, by observing the manner in which $\tan\delta$ changes as the applied voltage is varied.

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* The fact that a high value of power factor does not necessarily denote a correct value, has been emphasised by other authors (See Reference 7).

C. FLAT PLATE ELECTRODES WITH CONTROLLED LIQUID FILMS.

It has been seen in the earlier sections that in spite of the simplicity of flat plate electrodes, the presence and characteristics of air films lead to undesirable errors in the measurement of power factor. The uncertainty which attaches to these films suggested the replacement of air spaces by a liquid. It was considered that a freely flowing liquid film of controlled thickness between the electrode and the specimen might result in an intimate contact between the liquid and the confining surfaces, and that the characteristics of the composite dielectric so formed might be more predictable than in the case of a dielectric with air films. It was hoped that by this method an electrode system might be devised which combined the ease of application of flat plate electrodes with an accuracy greater than can be obtained with such electrodes.

It is interesting to consider what characteristics would be desirable in the liquid used for such a film. Equation (ii) can be re-written -

$$\tan\delta_D = \frac{C_c + C_D}{C_c}\tan\delta - \frac{C_D}{C_c}\tan\delta_c \qquad \text{.....(xix)}$$

In order that the measured value of power factor shall approximate to the true value, C_c should be as large and $\tan\delta_c$ as small as possible. Glycerine and ethyl alcohol suggested themselves on account of their high permittivities, but both were discarded after trial because of their high viscosity and

water absorbing power respectively. It is, however, quite possible that an inexpensive liquid hydrocarbon having a high permittivity may be available. For the tests about to be described, transformer oil was used, having a permittivity of about 2.2 .

It may be mentioned here that a method in which measurements are made on a composite dielectric for the purpose of determining the characteristics of one of the constituent dielectrics must normally be less satisfactory than a direct measurement. Such a method can only be justified if the advantages so gained, such as the ease of application of the electrode systems, the reduction in the uncertainty of contact effects, etc., balance the possibilities of increased experimental errors.

APPARATUS AND METHOD.

Some difficulty was experienced in designing an electrode system in which an oil film was maintained between the metal electrode surfaces and the specimen. Oil leaks proved troublesome as did the solution of the problem of introducing and withdrawing the oil. The method finally adopted was successful, and the general arrangement of the electrodes is shown in Fig.9 (c). Solid flat plate electrodes similar to those used in the earlier part of the investigation were provided with an outer knurled ring, screwed on its inner diameter, and engaging with a screw cut on the outer diameter of the electrodes. This ring was fitted with rubber and paper seals and when rotated, moved backwards and forwards along the central axis of the electrode. The requisite gaps for the liquid were obtained by the insertion of mica spacers. Oil was introduced into the gaps by means of the small tubes shown. Two similar tubes were provided diametrically opposite those mentioned above to allow for the egress of air. The electrodes were used with their plane surfaces vertical and the oil was introduced at the lowest point to prevent the formation of air locks.

The sequence of operations in assembling the specimen and electrodes was as follows.

The outer rings were first screwed back, away from the electrode surface. The electrodes were clamped to the

specimen after inserting the mica spacers to give the desired film thickness. The rings carrying the oil-seals were then screwed down until they met the surface of the specimen and the film spaces filled with oil. It was found that this series of operations could be performed in about five minutes.

EXPERIMENTAL RESULTS.

The results obtained with the oil film electrode system can be divided into two groups.

1. Measurements made on the 265 mil sample used in the previous tests, using films of thickness (each side) 3, 5 and 10 mils. The results are given in Figs. 31 and 32.

2. Measurements made on 3 samples each of glass, varnish paper board and ebonite, to investigate the behaviour of this electrode system under a wide range of operating conditions. The particulars of the samples and the experimental results are given in columns 1-8 of Table G, p.90. In addition, the power factor and permittivity of the oil were measured to provide the necessary information for the prediction of the power factor and permittivity of the dielectric from the values measured on the composite dielectric. The results are given in Fig.33.

GLASS

Thickness 265 mils

Oil gaps on L.V. and H.V. sides. Nominal values, in mils, indicated on curves

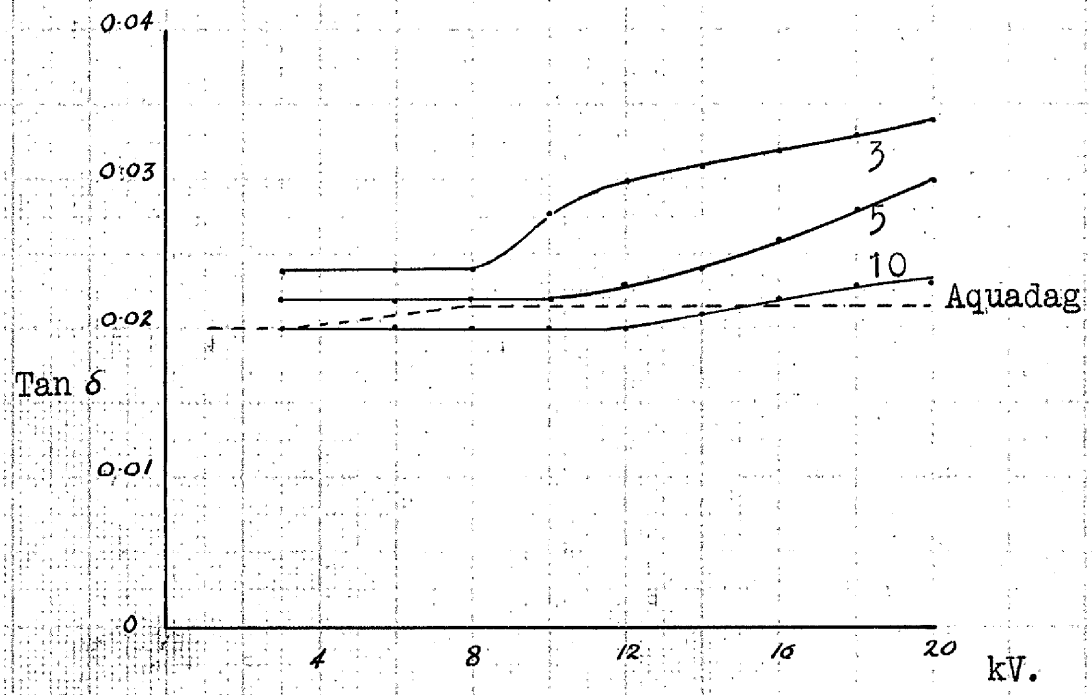

Fig. 31

GLASS

Thickness 265 mils

Oil gaps on L.V. and H.V. sides. Nominal values, in mils, indicated on curves.

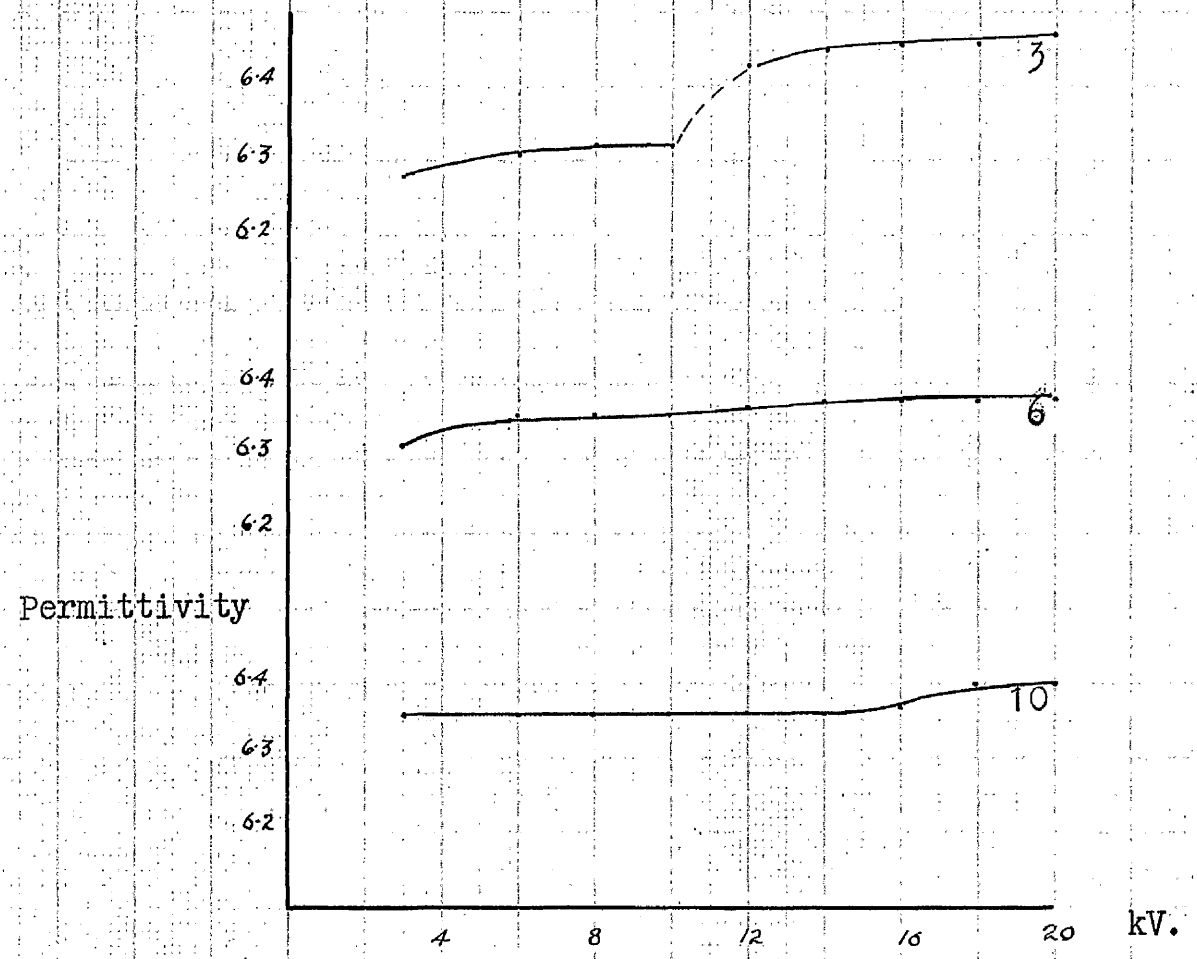

Fig. 32

CHARACTERISTICS OF OIL.

Measurements made on a film 0.22 in. thick, at room temperature. Permittivity constant at 2.22

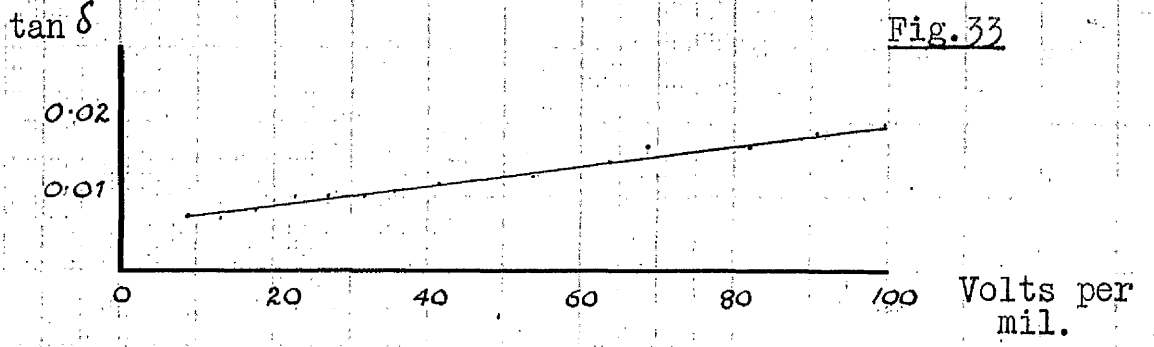

Fig.33

CALCULATED VOLTAGE GRADIENT ACROSS OIL FILM

of 20 mils used in series with solid insulation. Ratio given as Actual voltage-gradient / apparent overall gradient.

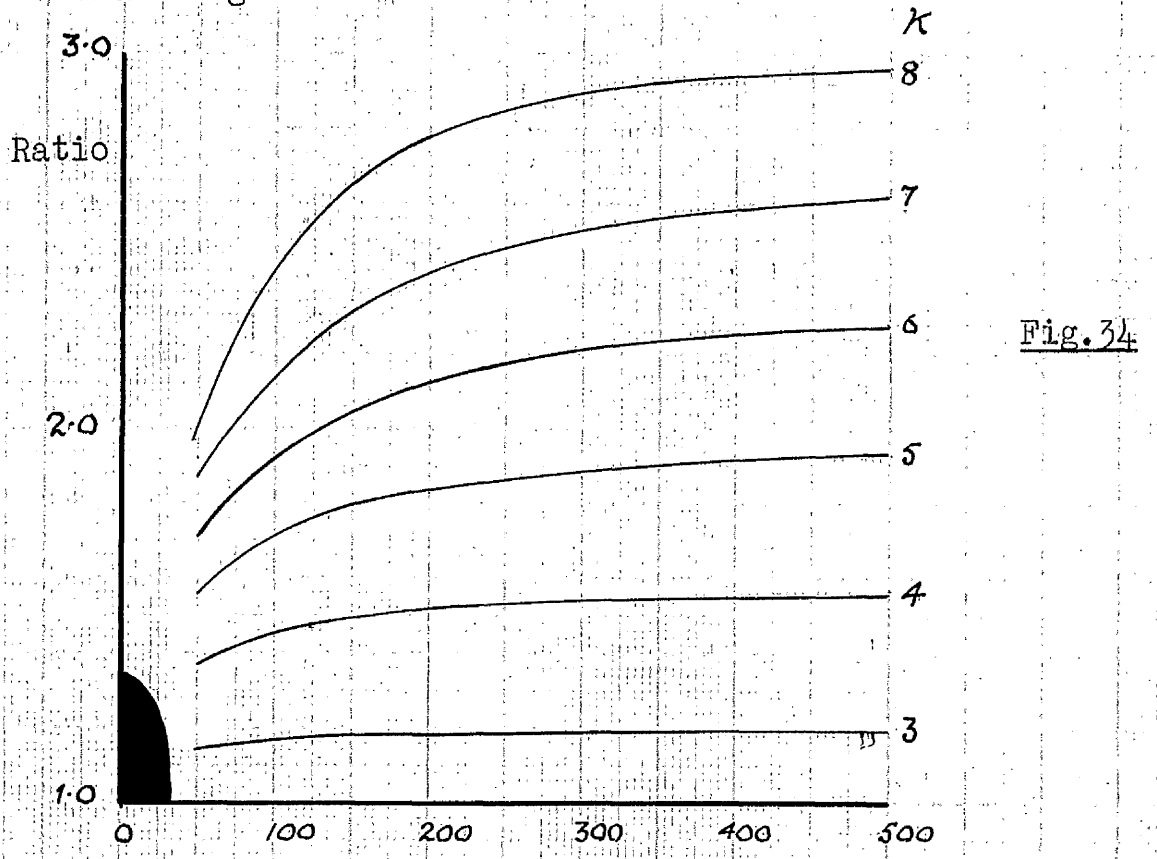

Fig.34

CONSIDERATION OF RESULTS.

Equation (xix) can be arranged as follows:-

$$\tan\delta_d = \frac{C_o}{C_o - C}\tan\delta - \frac{C}{C_o - C}\tan\delta_o \ldots\ldots\ldots (xx),$$

the suffix $_o$ referring to the total oil film. It is seen that $\tan\delta_d$ can be calculated from the overall measured value if C_o and $\tan\delta_o$ are known.

Fig. 33 shews that $\tan\delta_o$ is dependent on the voltage gradient existing in the oil. This gradient must therefore be considered in any calculation involving $\tan\delta_o$. Fig. 34 was accordingly constructed to give the ratio of the true gradient in the oil to the apparent gradient (based on the overall, specimen and films, thickness).

This ratio γ was calculated from

$$\gamma = \left(1 + \frac{t_d}{t_o}\right) \Big/ \left(1 + \frac{t_d K_o}{t_o K_d}\right) \quad \ldots\ldots\ldots (xxi)$$

The results shewn in Fig. 31 shew that with two 3 mil oil films the $\tan\delta$ /voltage curve and the K /voltage curve shew discontinuities with increasing voltage. The difficulty of producing a film of uniform thickness, even with a selected specimen may account for this behaviour. A smaller effect is observed with the 5 mil films and a negligible effect with the two 10 mil films. Since, with the thicker film, any surface irregularity in the specimen

has a smaller relative effect in the variation of film thickness, the remainder of the experimental work and the following discussion relate to the case of the two 10 mil films.

The value of C_o, calculated from the results given in Fig. 32 (2 x 10 mil film) by the relation

$$C_o = CC_D/(C_D - C) \qquad \text{.....(xxii)}$$

is 446 $\mu\mu F$, whence $K_o = 2.50$. The value of C_o, calculated from the value of $K_o = 2.22$ obtained from Fig.33, is 400 $\mu\mu F$. There is reason for some uncertainty in both values. $C_o = 446$ $\mu\mu F$ is calculated from the difference of two similar magnitudes C and C_D; $C_o = 400$ $\mu\mu F$ is calculated from the film thickness, and, as mentioned above, there is always the possibility of variation in this value. (It may be noted that the mean value of C_o calculated from C_D and C for each of the 9 samples in Table G gives 422 $\mu\mu F$).

The value of $\tan\delta_o$ must necessarily be taken from Fig.33, notwithstanding the fact that the characteristics of the oil when used as part of a composite dielectric may differ somewhat from its characteristics when used alone. The calculation of $\tan\delta_o$ from $\tan\delta$ and $\tan\delta_D$ is inadmissable since, as in the calculation of C_o from C and C_D,

a difference of two similar magnitudes is involved, but here each of the quantities is subject to greater inherent possibilities of error in measurement than the corresponding capacitance values. This is seen from a consideration of equation (viii) in the Appendix, which illustrates the much lower sensitivity of the Schering Bridge to a given percentage variation in power factor than to the same percentage variation in capacitance, and hence, for a given arrangement of bridge components, a greater uncertainty in the measured values of power factor than of capacitance.

By means of equation (xx), $\tan \delta_b$ was calculated for each of the 9 samples, using (a) $X_o = 2.22$ and (b) $X_o = 2.50$. The relevant figures are given in columns 15 and 16 of Table G. A specimen calculation is given below.

Calculation of $\tan \delta_b$ from measured values of $\tan \delta$ and C.

(1) Composite dielectric measurements.

Diameter of electrode.....11.5 cm.

Thickness of oil film.....2 x 10 mils.

Specimen and thickness.....Glass, 83 mils.

Voltage.....3 kV.

Measured values of $\tan \delta$ 0.040

C .....186 $\mu\mu$F.

(2) Aquadag electrode measurements.

Diameter of electrode.....11.5 cm

Voltage.....3 kV

Measured values of $\tan \delta_D$ 0.056

C_D.....340 $\mu\mu F$

Capacitance of oil film (total) based on

(a) $\chi_o = 2.22$,.....400 $\mu\mu F$

(b) $\chi_o = 2.50$,.....446 $\mu\mu F$

Permittivity of sample from

(1).....7.82

(2) (by equation xxii).....8.00

Stress factor, f, from Fig. 34,.....2.24

Overall voltage gradient ... $3000/(83+20)$ = 29 volts per mil.

Stress in oil ..... 29×2.24 = 65 volts per mil.

$\tan \delta_o$ at 65 volts per mil.....0.015

Calculated values of $\tan \delta_D$

(a) using $\chi_o = 2.22$.

$$\tan \delta_D = \frac{400}{400-186} \times 0.040 - \frac{186}{400-186} \times 0.015$$

$$= 0.075 - 0.013$$

$$= \underline{0.062}$$

(b) using $\chi_o = 2.50$.

$$\tan \delta_D = \frac{446}{446-186} \times 0.040 - \frac{186}{446-186} \times 0.015$$

$$= 0.069 - 0.011$$

$$= \underline{0.058}$$

1 Sample	2 Material	3 Thickness, mils	4 Voltage, kV	5 With aquadag	6 With aquadag	7 With oil	8 With oil	9 Permittivity	10 Permittivity	11 Overall Stress, v/mil	12 Stress Factor f	13 Stress in oil v/mil	14 $\tan\delta_o$	15 $\tan\delta_o$ calcd $\chi_o = 2.22$	16 $\tan\delta_o$ calcd $\chi_o = 2.50$	17 $\tan\delta_o$ measd	18 Remarks
				C_D μμF	$\tan\delta_D$	C μμF	$\tan\delta$	χ_D measd	χ_D calcd								
A	Glass	83	3	340	0.056	186	0.040	7.82	8.00	29	2.24	65	0.015	0.062	0.058	0.056	
			6	341	0.056	186.5	0.043	7.85	8.01	58	2.24	130	0.023*	0.060	0.057	0.056	Relatively thin sample of medium power factor
			10	342	0.057	187	0.048	7.87	8.02	97	2.24	217	+	–	–	–	
			14	343	0.059	187.5	0.053	7.89	8.03	136	2.24	320	+	–	–	–	
B	Glass	121	3	243	0.046	154	0.034	8.14	8.40	21	2.56	54	0.013	0.047	0.045	0.046	
			6	243	0.048	154	0.034	8.14	8.40	43	2.56	108	0.021	0.040	0.041	0.048	Effect of second term of(××) increasing as $\tan\delta_o$
			10	244	0.052	154	0.038	8.17	8.40	71	2.56	181	+	–	–	–	increases
			14	244	0.052	155	0.041	8.17	8.50	99	2.56	253	+	–	–	–	
C	Glass	265	3	102	0.020	83	0.020	7.50	7.70	10.5	2.70	28	0.010	0.023	0.023	0.020	
			6	102	0.022	83	0.020	7.50	7.70	21	2.70	56	0.014	0.021	0.021	0.022	
			10	102	0.023	83	0.021	7.50	7.70	35	2.70	94	0.018	0.022	0.022	0.023	Relatively thick sample of medium-low power
			14	102	0.023	83	0.026	7.50	7.70	49	2.70	132	+	–	–	–	
			18	102	0.023	83	0.030	7.50	7.70	63	2.70	170	+	–	–	–	factor
D	Varnish Paper Board	130	3	198	0.160	135	0.110	7.12	7.35	20	2.30	46	0.012	0.160	0.153	0.160	
			6	198	0.160	135	0.110	7.12	7.35	40	2.30	92	0.018	0.157	0.150	0.160	Relatively thin sample of high power factor
			10	199	0.160	136	0.110	7.17	7.42	67	2.30	154	+	–	–	–	
			14	205	0.180	136	0.112	7.37	7.42	93	2.30	214	+	–	–	–	
E	Varnish Paper Board	259	3	114	0.220	90	0.179	8.17	8.32	11	2.90	32	0.010	0.228	0.221	0.220	
			6	114	0.220	90	0.179	8.17	8.32	22	2.90	64	0.014	0.227	0.220	0.220	Relatively thick sample of high power factor
			10	115	0.220	90	0.179	8.25	8.32	36	2.90	104	0.020	0.225	0.219	0.220	
			14	116	0.223	90	0.179	8.32	8.32	50	2.90	145	+	–	–	–	
F	Varnish Paper Board	511	3	48.9	0.114	43.7	0.100	6.91	7.02	5.7	2.58	14.7	0.008	0.111	0.110	0.114	
			6	48.9	0.114	43.7	0.100	6.91	7.02	11.3	2.58	29	0.010	0.111	0.110	0.114	Thick sample of high power factor: the most favourable conditions.
			10	48.9	0.116	43.7	0.101	6.91	7.02	19	2.58	49	0.012	0.112	0.111	0.116	
			14	48.9	0.116	43.7	0.103	6.91	7.02	26	2.58	67	0.015	0.114	0.112	0.116	
			18	49.1	0.117	43.8	0.105	6.95	7.02	34	2.58	88	0.018	0.116	0.115	0.117	
G	Ebonite	16	3	678	0.014	260	0.010	3.00	3.30	83	1.08	90	0.018	-0.004	-0.001	0.014	
			4	680	0.014	260	0.010	3.01	3.30	111	1.08	120	0.022	-0.012	-0.024	0.014	Thin sample of low power factor: the least favourable conditions
			6	684	0.014	260	0.012	3.03	3.30	167	1.08	180	+	–	–	–	
			8	686	0.014	260	0.019	3.04	3.30	222	1.08	240	+	–	–	–	
H	Ebonite	40	3	270	0.009	165	0.010	2.99	3.12	50	1.14	57	0.013	0.008	0.008	0.009	
			6	270	0.010	165	0.010	2.99	3.12	100	1.14	114	0.021	0.002	0.004	0.010	Relatively thin sample of low power factor
			8	270	0.010	165	0.011	2.99	3.12	133	1.14	152	+	–	–	–	
			10	270	0.010	165	0.012	2.99	3.12	167	1.14	190	+	–	–	–	
I	Ebonite	66	3	160	0.010	119	0.010	2.93	3.09	35	1.15	40	0.012	0.009	0.010	0.010	
			6	160	0.010	119	0.010	2.93	3.09	70	1.15	80	0.017	0.007	0.008	0.010	Relatively thin sample (but thicker than H) of low permittivity and power factor.
			10	160	0.011	119	0.010	2.93	3.09	116	1.15	133	+	–	–	–	
			12	160	0.013	119	0.012	2.93	3.09	140	1.15	161	+	–	–	–	

* Estimated
\+ Beyond measured values

TABLE G

Good agreement is observed between the values of $\tan\delta_D$ measured by means of aquadag electrodes and those calculated from C and $\tan\delta$, in the case of samples which are relatively thick and/or of relatively high power factor. The method fails completely in the case of one very thin sample (G) having a low power factor. Equation (xx) shows that when $\tan\delta_o \to 0$, or when $\tan\delta_D \gg \tan\delta_o$ the measured value of power factor is $\frac{C}{C_o} \times 100\ \%$ low. The influence of the 2nd term of (xx) increases as $\tan\delta_o$, K_D, t_o increase and as K_o, t_D and $\tan\delta_D$ decrease. These effects are seen in the good agreement between the measured and calculated values of $\tan\delta_D$ in the case of samples, A, C, D, E, F and I, the fair agreement in the cases of samples B and H at low voltages and the complete lack of agreement in the case of Sample G.

It would seem that in those cases where agreement is worst, the reason lies in the high value of the second term in (xx) compared with the first term. Under such conditions any uncertainty in the value of $\tan\delta_o$ which should be used leads to large variations in $\tan\delta_D$. Thus any differences in $\tan\delta_o$ when the oil is used alone and when used as part of a composite dielectric assume an increased importance, as does the fact, otherwise of little importance, that in the calculation of γ (xxi) no allowance was made for the power factors of the film and specimen.

It may be said that in general good agreement is shown between the measured values of $\tan \delta_D$ and the values calculated by means of the composite dielectric method. The measured and calculated values of χ_D show an even better agreement.

In retrospect, it appears that for improved accuracy by this method, $\frac{\tan \delta_o}{\tan \delta_D}$ should be as small as possible, and $\frac{C_o}{C_D}$ should be as large as possible consistent with a film of reasonable thickness.

CONCLUSIONS.

1. The experiments relating to flat-plate electrodes with the normal and additional air films resulted in interesting evidence concerning the behaviour of such films but led to no improved way of employing these electrodes. When such electrodes are used, they should be applied to the specimen under considerable mechanical pressure and the applied voltage should be below that at which the discontinuity in the power factor/voltage curve occurs.

2. The use of tinfoil mounted on rubber discs and applied to the specimen without an oil adhesive proved no more satisfactory than the use of flat-plates. Since in this case it is probable that average thickness of the air film is as small as can be obtained by any means, it may be concluded that the development of a satisfactory electrode system employing removable, dry, metal surfaced electrodes is not very promising.

3. Tinfoil electrodes applied by means of a trace of oil are satisfactory. Apart from the time taken in applying them they probably represent one of the best methods for industrial measurements of power factor and permittivity. The attempt to reduce this time by applying the tinfoil mounted on a fabric base was successful but the resulting electrode was easily damaged.

4. The development of electrodes using controlled oil films gave encouraging results. It is suggested that this electrode system justifies further consideration, particularly by the investigation of other and more suitable liquids for the film.

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APPENDIX.

The Sensitivity of the Schering Bridge Network.*

The sensitivity required of a bridge network depends on the particular measurement to be made. Measurements of very high precision have been made by suitable arrangements of this network (loss angle to 1 second and capacitance to 1 part in 10^6). In many measurements such precision is not necessary; it is desirable however, to consider the sensitivity of a particular network for a proper appreciation of the values of permittivity and loss angle, and in particular the changes in these quantities, measured thereby. That is, it is desirable to know how given changes in each of the network components affect the detector current.

Lord Rayleigh[+] dealt with the general theory of sensitivity of bridge networks, and the particular case of the Schering bridge network has been considered by Hallo and de Zoeten[‡], Tschiassny[⊕], Miller[⋕], and Alten[§]. The following discussion gives a simple method of arriving at the conditions of sensitivity. It is found that the conclusions are in agreement with those in the papers mentioned.

If, in the bridge network shown in Fig.35 the impedances z_5 and z_6 are fixed, and z_1 is to be measured, then the values of z_2, z_3, z_4, can be deduced in terms of z_1, z_5 and z_6

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*Published in the Journal of the Royal Technical College, 1937; See Reference (30).
+See Reference (31)
‡Ibid (32)
⊕Ibid (33)
⋕Ibid (34)
§ Ibid (35)

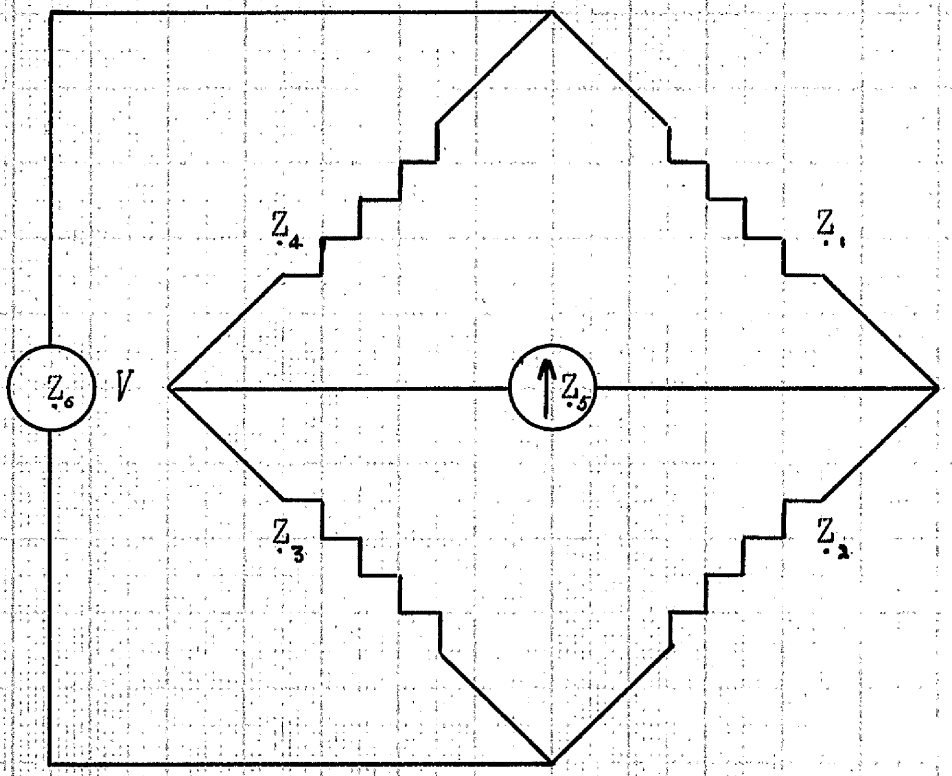

Fig.35

for maximum sensitivity. In most networks, the particular conditions imposed by the measurement render the application of these ideal values impossible. In the simplest form of the high-voltage Schering bridge network, shown in Fig. 6, one characteristic condition imposed is the great divergence (e.g. x 10^5) between the order of magnitude of the impedances Z_1 and Z_4, and the other impedances. This results from the necessity of confining the high voltage to the branches Z_1, Z_4. The current in the detector branch is given* by Maxwells cyclic current method as -

$$I = V \frac{Z_1 Z_3 - Z_2 Z_4}{\Delta} ,$$

where $$\Delta = \begin{vmatrix} -Z_5 & -(Z_3+Z_4) & (Z_3+Z_4+Z_6) \\ (Z_2+Z_3+Z_5) & (Z_2+Z_3) & -Z_3 \\ -Z_5 & (Z_1+Z_4) & -Z_4 \end{vmatrix} .$$

By inspection of the determinant, the only terms involving the product $Z_1 Z_4$ are seen to be $Z_1 Z_4 Z_2, Z_1 Z_4 Z_3$ and $Z_1 Z_4 Z_5$. By comparison with these, all other terms may be neglected, whence

$$I = V \frac{Z_1 Z_3 - Z_2 Z_4}{Z_1 Z_4 (Z_2+Z_3+Z_5)}$$

$$= V \frac{\frac{Z_3}{Z_4} - \frac{Z_2}{Z_1}}{(Z_2+Z_3+Z_5)} \qquad \ldots\ldots\ldots (i)$$

---

* See Reference (6b), p.51.

where

$$Z_1 = Z_1 \angle \phi_1 = \sqrt{1+\omega^2 C_1^2 r^2}\,/\,\omega C_1 \;\angle \tan^{-1} 1/\omega C_1 r$$

(C_1, r being the effective series capacitance and resistance of the test specimen).

$$Z_2 = Z_2 \angle \phi_2 = R_3 \angle 0$$

$$Z_3 = Z_3 \angle \phi_3 = R_4 \Big/ \sqrt{1+\omega^2 C_4^2 R_4^2} \;\angle \tan^{-1} \omega C_4 R_4$$

$$Z_4 = Z_4 \angle \phi_4 = 1 \Big/ \omega C_5 \angle -\pi/2$$

$$Z_5 = Z_5 \angle \phi_5 = \sqrt{R_5^2 + \omega^2 L_5^2} \;\angle \tan^{-1} \omega L_5 / R_5$$

At balance,

$$Z_3/Z_4 = Z_2/Z_1$$

and

$$\phi_3 - \phi_4 = \phi_2 - \phi_1 \quad *$$

The magnitude of the detector current due to changes in the branch components from balance conditions will therefore be proportional to

$$\left[Z_3/Z_4 - Z_2/Z_1 \right]$$

$$= \left[\frac{\omega R_4 C_5}{\sqrt{1+\omega^2 R_4^2 C_4^2}} - \frac{\omega R_3 C_1}{\sqrt{1+\omega^2 r^2 C_1^2}} \right],$$

and may be written

$$I = V \frac{\left[\frac{\omega R_4 C_5}{(1+\omega^2 R_4^2 C_4^2)^{\frac{1}{2}}} - \frac{\omega R_3 C_1}{(1+\tan^2\delta)^{\frac{1}{2}}} \right]}{\left[(R_3+R_4+R_5)^2 + \omega^2 L_5^2 \right]^{\frac{1}{2}}} \qquad \ldots\ldots\ldots (ii)$$

---

* whence, using the nomenclature of Fig.6, $(r - j/\omega C_1) = R_3 (C_4/C_5 - j/\omega C_5 R_4)$; from which $C_1 = R_4 C_5 / R_3$ and $\tan\delta = \omega R_4 C_4$.

since $\omega C_1 r = \tan\delta = \omega R_4 C_4$ at balance.

If $V \omega C_1 R_3 / \left[(R_3 + R_4 + R_5)^2 + \omega^2 L_5^2 \right]^{\frac{1}{2}}$ be denoted by a, the changes in I due to small changes in R_3, C_4, C_1, C_5 and $\tan\delta$ are:

$$\delta I_{R_3} = \delta R_3 \cdot a / R_3 \qquad \ldots\ldots (iii)$$

$$\delta I_{C_4} = \delta C_4 \cdot a \omega R_4 \qquad \ldots\ldots (iv)$$

$$\delta I_{C_1} = \delta C_1 \cdot a / C_1 \qquad \ldots\ldots (v)$$

$$\delta I_{C_5} = \delta C_5 \cdot a / C_5 \qquad \ldots\ldots (vi)$$

$$\delta I_{\tan\delta} = \delta (\tan\delta) \cdot a \qquad \ldots\ldots (vii)$$

From (iii) and (iv), for the same change in I, $\delta R_3 / \delta C_4 = \omega R_3 R_4$, and the relative percentage changes of R_3 and C_4 are given by

$$\omega R_3 R_4 \times C_4 / R_3 = \omega R_4 C_4 = \tan\delta \qquad \ldots\ldots\ldots (viii)$$

Similarly, from (v) and (vii) the ratio of the percentage changes of C_1, and $\tan\delta$ for the same change in I is given by

$$C_1 \times (\tan\delta)/C_1 = \tan\delta \qquad \ldots\ldots\ldots (ix)$$

It is seen from (ii) that the sensitivity is proportional to the operating voltage and to the frequency. The latter is commonly fixed at 50 in commercial bridges, but the range of operating voltage which may be met with in a particular set of measurements is often large, with a proportionate variation of sensitivity available for the determination of C_1 and $\tan\delta$.

The sensitivity also increases with decrease of the detector impedance, but this increase, will in general be restricted, due to the preponderating effect of R_3 and R_4.

Increase of R_4 (with C_s fixed) produces an improvement of sensitivity in respect of changes of C_4, $\tan\delta$ or C_1, but not of R_3, the improvement becoming smaller as the effect of the R_3 and R_4 terms in the denominator increases. R_4 is, for convenience, often fixed at a value $1000/\pi\ \Omega$ so that $\tan\delta = \omega R_4 C_4 = 0{\cdot}1\ C_4$ (C_4 in μF).

Increase of C_1 (with C_s, R_4 fixed) increases the sensitivity in respect of changes in R_3, and increase of C_s (with C_1, R_4 fixed) in respect of changes in C_4 and $\tan\delta$. Constructional limitations of high-voltage air capacitors usually result in capacitances of the order of 100 $\mu\mu F$ for C_s.

From (viii) and (ix), it is seen that to produce similar changes in the detector current, the ratio of the percentage changes of R_3 and C_4 or of C_1 and $\tan\delta$ is equal to $\tan\delta$. Thus, with a balance of 4 kV where $C_1 = 102.1\ \mu\mu F$, $C_s = 106\ \mu\mu F$, $C_4 = 0.10\ \mu F$, $\tan\delta = 0.010$, $R_3 = 330.8\ \Omega$ and $R_4 = 318.4\ \Omega$, a change of 1 per cent in either $\tan\delta$ or C_4 will produce the same change in I as a change of 1/100 per cent in C_1 or R_3. At 40 kV the corresponding changes would be 1/10 per cent and 1/1000 per cent. Alternatively,

for the smallest detectable value of I, C_1 can be measured with 100 times the precision of $\tan \delta$.

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